A STATISTICAL STUDY OF SPECTROMETRIC OIL ANALYSIS DATA FROM THE NAVAL OIL ANALYSIS PROGRAM

by

John Patrick Riceman

United States Naval Postgraduate School

THESIS

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John Patrick Riceman
Captain, United States Army
B.S., United States Military Academy, 1963

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ABSTRACT

This thesis examines spectrometric oil analysis data from two sources in an attempt to formulate a statistical model which will be useful in monitoring aircraft engines in the Naval Oil Analysis Program. Initially, experimental data, gathered for an Air Force study, is used to determine if the measurement error inherent in the monitoring procedure is normally distributed and if correlations exist between measurements for different wear metals. Based on the results of this investigation, a study is made of operational data from Wright reciprocating engines of the R1820-82 model type. This investigation leads to the conclusion that a multivariate regression model is useful in estimating the parameters of the distribution of analyses from properly operating engines of this type. A procedure is then suggested which would employ the readings from past oil analyses from a particular engine to determine its present condition.

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I. INTRODUCTION

For several years the technique of using the spectrometric analysis of oil samples as an aid in determining the condition of diesel engines has been employed successfully by major railroads and various other users of large diesel equipment. In 1956, a trial program was begun at the Naval Air Rework Facility in Pensacola to determine if this method could also be used to monitor aircraft engines. Since that time the program has proved successful and has evolved into the Naval Oil Analysis Program (NOAP). It is planned that this program will eventually include all Navy fluid lubricated mechanical systems. A more detailed history of the program is contained in Refs. 1 and 2.

Since this thesis is concerned with an investigation of data collected at the Pensacola laboratory, the following descriptions of the operation will be limited to the procedures used there. Reciprocating aircraft engines are sampled approximately every 30 hours. The sample is taken after the aircraft has returned from a flight and before the oil has become cold. It is immediately sent to the laboratory by air mail and is analyzed on the day received. The analysis is accomplished by a spectrometer using the rotating graphite electrode technique. Measurements of the parts per million (ppm) content of ten metallic elements, which might be indicative of engine wear, are made

simultaneously. Of these ten, aluminum, chromium, iron, silver, copper, magnesium and nickel are those which are relevent to engines of the model considered in this report. The ppm readings are automatically recorded on a punched card which also contains various other hand-entered data identifying the sample.

Once the data has been recorded, it is used to aid in determining what the operating condition of the engine might be. Presumably, if the engine is operating properly, the amount of metallic contamination in the circulating oil should be within certain normal limits. In addition, it is felt that the amount of contamination added to the oil since it was last sampled should be within specific limits if the engine is in good working condition. If, however, the engine is discrepant and excessive wear is present, this will presumably cause an abnormal addition of metallic contaminates to the circulating oil.

Thus, when a sample has been analyzed and the results recorded, both the magnitudes of the present readings and the changes in the readings since the last sampling are compared with threshold limits which have been developed for each engine type and each metallic element relevent to that engine. If the results fall outside the prescribed limits, some action is generally taken by the laboratory. Usually another sample is requested and the previous results are verified. If the abnormality persists, either

the aircraft is grounded for maintenance or future samples are taken more frequently than the usual 30 hour interval.

At present, these threshold limits are subjectively set and vary only from element to element and among engine model types. They are based on the past history of the aircraft model which includes the data supplied by the engine manufacturer before the model is placed in service and experience accumulated once the model is in use. The limits are not used as sharp boundaries for classifying engines as normal or discrepant but merely as indicators upon which a subjective decision as to the action to be taken can be based.

This report examines two sets of data from the Pensacola laboratory with the intention of determining the propriety of three assumptions implicit in this classification procedure. Since the same threshold limits are used for all engines of a particular model, it is assumed that all normally operating engines of the same type can be expected to have the same amounts of metallic contamination in their oil systems. In addition, since threshold limits are constant for a given element and model type, variations in other factors, such as the operating hours since the last oil change, must be ignored or subjectively introduced into the classification procedure. Finally, since threshold limits are set for each element independent of the limits for other elements, readings for different elements are assumed to be uncorrelated.

Once these assumptions are verified or rejected, a statistical model is formulated to aid in establishing a more objective classification criterion.

II. ERRORS INHERENT IN THE MONITORING PROCEDURE

Since the intention of NOAP is to make inferences about the condition of aircraft engines, based on the amount of wear metal contamination in the engine's oil system, it is extremely important that the amount of contamination recorded at the laboratory accurately reflect the actual amount present in the engine. For the purposes of this report, measurement error will be defined as the difference between the parts per million content of an element recorded as present in a particular engine at a point in time and the actual content at that time. In NOAP there are a variety of potential sources of error, all of which can contribute to the net measurement error defined above.

A. ERRORS IN SAMPLING

As was mentioned earlier, oil samples are taken from reciprocating engines normally every 30 flying hours and while the oil is still hot. This sampling is accomplished with a special sampling kit consisting of a sampling tube and a sampling bottle. The tube is inserted into the oil reservoir, and when it has filled the top end is stopped with the operator's finger. The contents are then transferred to the bottle, which is immediately forwarded to the laboratory for analysis. When the sample is analyzed, a small portion of the oil in the bottle is used in the

analysis [Refs. 1 and 2]. Thus, an extremely small amount of oil is used to determine the extent of contamination in the engine's entire oil system. Any lack of homogeneity in the engine's oil reservoir will result in a non-representative sample. Further, any contamination added to the sample through a lack of cleanliness of the sampling tube and bottle or through handling at the laboratory will contribute to the measurement error.

B. ERRORS IN RECORDING

At the time the sample is taken, certain data including the date, the operating hours since the last oil change, the hours since the last overhaul of the engine, the engine serial number and the model number are recorded and mailed to the laboratory with the sample. Various portions of this data are transferred from other records. At the laboratory the data are entered by hand on the permanent record cards maintained there [Ref. 1]. This entire sequence of recording and transferring data from one record to another can result in mistakes.

Unfortunately, as with the errors in sampling, there is no data available at the present time that can be used to measure this error.

C. ERRORS IN ANALYSIS

When an oil sample is received at the Pensacola laboratory, it is analyzed using a direct reading spectrometer with spark excitation, stationary and rotating disc

electrodes. The sample bottle cap is filled with oil and placed in the spark stand. The gap between the two electrodes is set and the disc electrode begins to rotate at 30 rpm. As the electrode rotates, a thin film of oil is forced to the area under the fixed electrode. A high energy spark is then fired across the gap and the film of oil is burned for 25 seconds. The light from the burning oil is separated so that its intensity at the wave lengths, produced by the elements to be analyzed, can be compared with built-in standards. The average intensity over the burning period is then measured for each element simultaneously and converted into parts per million. These readings are automatically recorded on the engine history card [Ref. 2].

If it is assumed that there were no errors in the sampling or recording and thus, that the oil used in the analysis is representative of the oil in the engine, any difference between the true content of contamination in the engine and that recorded after the analysis can be attributed to an analysis error. An experiment designed to measure the effects of this type of error has been conducted and the results of an analysis of this data are presented in the next section.

III. DISCUSSION OF THE ANALYSIS ERROR

Although the error due to the spectrometric analysis of the oil is not the only possible source of error, it certainly is a major contributor to the over-all measurement error defined earlier. For this reason, an examination of data accumulated for a study conducted by the Air Force [Ref. 3] was performed and the results are discussed in this section.

A. DATA

In 1967, the Pensacola laboratory participated in an experiment conducted by the Air Force. During a 30 day period the laboratory received 100 oil samples. These were to be analyzed in the normal manner and the results reported. Although the laboratory was not aware of it, these 100 samples consisted of ten samples each repeated ten times. Thus, the laboratory actually repeated the analysis of ten different samples ten times. The results of the analyses, for the seven elements of interest in this report, are included in the Computer Output section, where the readings on like samples are in groups numbered from one to ten. Missing data accounts for some groups having less than ten readings.

For each of the ten groups of repetitious analyses, the sample mean and standard deviation were calculated for each element. If, for example, X_i is the $i^{\underline{th}}$ reading for

aluminum in a group of size n, then the sample mean, $\bar{X}$, and standard deviation, S, for aluminum are

$$X = \left(\sum_{i=1}^{n} X_i\right)/n$$

and

$$S = \left[\sum_{i=1}^{n} (X_i - \bar{X})^2/(n-1)\right]^{\frac{1}{2}} \tag{1}$$

respectively. The results of these calculations are also presented in the Computer Output section for each element and each group of samples.

In addition, for each sample group, an estimated correlation matrix, $\underline{R}$, was calculated, where if r_{ij} is the element in the $i^{\underline{th}}$ row and $j^{\underline{th}}$ column of $\underline{R}$, then

$$r_{ij} = \frac{\sum_{k=1}^{n} (X_{i,k} - \bar{X}_i)(X_{j,k} - \bar{X}_j)}{\left[\sum_{k=1}^{n} (X_{i,k} - \bar{X}_i)^2 \sum_{k=1}^{n} (X_{j,k} - \bar{X}_j)^2\right]^{\frac{1}{2}}} \tag{2}$$

where $X_{i,k}$ is the $k^{\underline{th}}$ reading for the $i^{\underline{th}}$ element, and $\bar{X}_i$ is the element's sample mean. These correlation matrices are included in the Computer Output section.

These preliminary computations provided statistics which were used to test certain hypotheses concerning the probability distribution of the analysis error.

B. TEST FOR NORMALITY

Since the overall measurement error is the net effect of errors arising from a variety of sources, the Central Limit Theorem of Probability Theory [Ref. 4] provides good justification for making an assumption of normality in the distribution of this error. Thus, any additional evidence, which tends to point to the normality of one of the contributing sources of error, will serve to strengthen the overall assumption. The experiment conducted by the Air Force provided data which was used to test for normality in the distribution of the analysis error.

If $\underline{X}_k$ is defined as a seven-component vector of sample readings arising from sample group $k = 1,2,\ldots,10$, then

$$\underline{X}_k = \underline{\mu}_k + \underline{e}_k$$

where $\underline{\mu}_k$ is a seven-component vector of the true metallic content of seven elements in the sample associated with the $k^{\underline{th}}$ group of readings, and $\underline{e}_k$ is the seven-component random analysis error vector. Thus, if it is assumed that $\underline{e}_k$ is a multivariate normal random variable with zero mean vector and unknown covariance matrix $\underline{\Sigma}_k$, then $\underline{X}_k$ is a multivariate normal random variable with mean vector $\underline{\mu}_k$ and covariance matrix $\underline{\Sigma}_k$, denoted $N(\underline{\mu}_k, \underline{\Sigma}_k)$.

If this assumption is correct, it is possible to perform a transformation of the form,

$$\underline{Z}_k = \underline{P}_k(\underline{X}_k - \underline{\mu}_k),$$

which will produce a multivariate normal random variable $\underline{Z}_k$ which has mean vector zero and covariance matrix I, the identity matrix. For details of this transformation see Appendix A.

For each of the ten groups of sample readings the mean vector $\underline{\mu}_k$ was estimated using

$$\overline{\underline{X}}_k = \sum_{j=1}^{n_k} \underline{X}_{kj}/n_k \tag{3}$$

where $\underline{X}_{kj}$ is the $j^{\underline{th}}$ vector of sample readings from the $k^{\underline{th}}$ sample group, and n_k is the number of readings in that group. In addition, the covariance matrices, $\underline{\Sigma}_k$, were estimated using the unbiased estimator

$$\hat{\underline{\Sigma}}_k = (S_i \; S_j \; r_{ij})_k \tag{4}$$

where S_i is the estimated standard deviation for the $i^{\underline{th}}$ element, computed as in equation (1), and r_{ij} is as defined by equation (2). For each of the ten groups the non-singular matrix, $\underline{P}_k$, was found and the transformation,

$$\underline{Z}_{k,i} = \underline{P}_k(\underline{X}_{k,i} - \overline{\underline{X}}_k),$$

performed on each vector of readings, $\underline{X}_{k,i}$, in the $k^{\underline{th}}$ group, $k = 1,2,\ldots,10$. In this way vectors $\underline{Z}_{k,i}$ were produced, the components of which are stochastically independent and are distributed according to N(0,1) if the hypothesis is true.

All readings from the ten groups were then pooled to produce a sample of 651 deviates, assumed to be univariate normal. The Kolmogorov-Smirnov goodness of fit test was applied to this sample and the resulting test statistic of .0215 was not significant even at the .20-level. Thus, the hypothesis of normality in the distribution of the analysis error was accepted. For details of the test see Appendix A.

C. TEST FOR EQUALITY OF COVARIANCE MATRICES

Let $\underline{X}_k$ again be defined as

$$\underline{X}_k = \underline{\mu}_k + \underline{e}_k$$

as in the previous section, where now $\underline{e}_k$ is assumed to be a multivariate normal random vector. In addition, let the estimators $\overline{\underline{X}}_k$ and $\hat{\underline{\Sigma}}_k$ be defined by equations (3) and (4) respectively. Then, the Air Force data can be used to produce ten estimated covariance matrices $\hat{\underline{\Sigma}}_k$, each associated with a different sample group and thus, a different true content vector $\underline{\mu}_k$. If the true covariance matrices, $\underline{\Sigma}_k$, are independent of the vector $\underline{\mu}_k$, and thus constant for all k = 1,2,...,10, the ten estimated matrices could be pooled to obtain an over-all estimate of $\underline{\Sigma}$. This hypothesis of the equality of the ten covariance matrices was tested, and the results led to the rejection of the hypothesis at the .10 level of significance. The details of the test used and the results obtained are included in Appendix A.

In an attempt to account for the apparent variability among covariance matrices from different sample groups, a regression model was formulated. It had been suggested by Baird-Atomic Inc., the manufacturer of the spectrometer used at the Pensacola laboratory, that the variability in readings for a given element is dependent on the true content of the element. Specifically, the relationship is assumed to be

$$\sigma^2 = a + b\mu^2$$

where σ^2 is the variance of repeated analyses of the same sample for a given element, μ is the true parts per million content of the element, and a and b are constants [Ref. 5]. The model,

$$S^2 = a + b\overline{X}^2 + e,$$

was used to examine the propriety of this relationship for each of the seven elements under consideration. Here S^2 is the square of the standard deviation estimate defined by equation (1), $\overline{X}^2$ is the square of the sample mean, and e is a random variable. For each element, a and b were estimated using least-squares techniques and, under the assumption that variations about the regression line are normally distributed, the hypothesis, b = 0, was tested [Refs. 6 and 7]. A t-test was used and the significance level set at .10. Of the seven slopes tested, those associated with aluminum, iron, copper and magnesium were significantly

non-zero. It should be mentioned that the true content of the other elements did not vary much among the ten sample groups. The results of the least-squares estimation, together with the numerical results of the t-tests, are included in the Computer Output section.

D. TEST FOR INDEPENDENCE AMONG ELEMENTS

Because of the apparent dependence of the variance of repeated readings upon the true content of an element, it was felt that covariances between elements might depend upon the content of the elements concerned. If this were the case, then, for example, a particularly high reading of one element might be "explained" by a corresponding low reading of another element. Under the assumption of normality, the hypothesis of independence is equivalent to the hypothesis of zero correlation. This hypothesis was tested, using each of the ten correlation matrices $\underline{R}$, defined by equation (2). Of the ten tests conducted, only three of them were not significant at the .10 level. The numerical results and the test used are given in Appendix A. It can be noted from the correlation matrices in the Computer Output section that particularly strong correlations seem to exist between iron and copper, silver and copper, and magnesium and iron.

Thus, it appears that, in general, the readings of different elements are not independent, and some explanation, for example, of an erroneous copper reading may come

from an examination of the corresponding iron reading on the same sample.

IV. ANALYSIS OF OPERATIONAL DATA

Using the evidence provided by the Air Force data to support the assumption of normality in the distribution of the measurement error, an investigation of some operational data was conducted. A statistical model, which makes use of the apparent correlations between the readings on different elements, was formulated. The details of the model's formulation are discussed in this section.

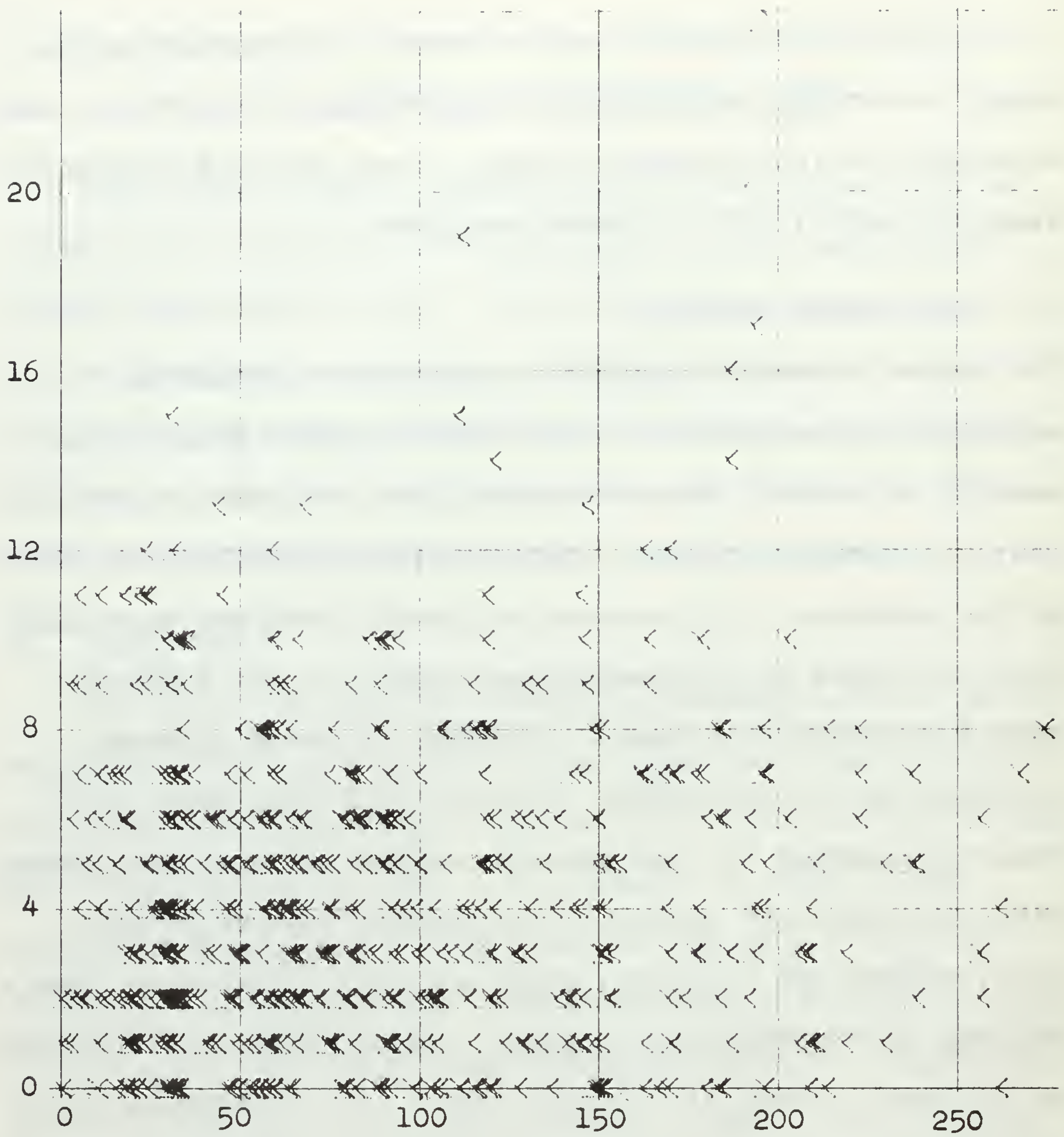
A. NOAP DATA

The Naval Air Rework Facility at Pensacola provided a magnetic data tape containing the records of operational analyses performed there from the beginning of July to the end of September in 1967. The records of some 21,000 different analyses were included on the tape. For each analysis the engine model number and serial number, as well as the date the analysis was performed and its results in parts per million for each relevant element, are recorded. In addition, it includes the operating hours since the last oil change and since the last overhaul of the engine for each sample. Unfortunately, the action recommended by the laboratory after each sample was analyzed and the results of that action were not available with the tape. For this reason, there was no way of determining with certainty which analyses were on oil from properly operating engines and which were from discrepant engines.

Of the 113 different engine models represented on the tape, the Wright reciprocating engine model, R1820-82, was selected for investigation since it was the most frequently sampled, with 4,134 different analyses.

B. PRELIMINARY RESULTS

Since it seemed logical to expect the content of metallic contamination to show some increase from normal wear in a properly operating engine as the hours since the last oil change increase, some preliminary plots were made by the computer. Six hundred different analyses were used with no regard to the particular engine of the R1820-82 type from which they came. For each of seven elements, relevant to the monitoring of engines of this type, the computer plotted the ppm content versus the operating hours since the last oil change. For three of the elements, iron, copper and aluminum, there was some indication that a buildup of contamination occurs. The other four plots gave no evidence of any significant trend. For comparison, the plot of chromium is included with those of aluminum, iron, and copper in Figures 1 to 4, respectively. Since these plots were made on sample readings from a variety of engines, they did not indicate whether a particular engine can be expected to show the same trend. For this reason, the five most frequently sampled engines of the R1820-82 type were selected and the computer was again used to plot the data from these engines. Five different symbols,



X-SCALE = 50 UNITS PER INCH

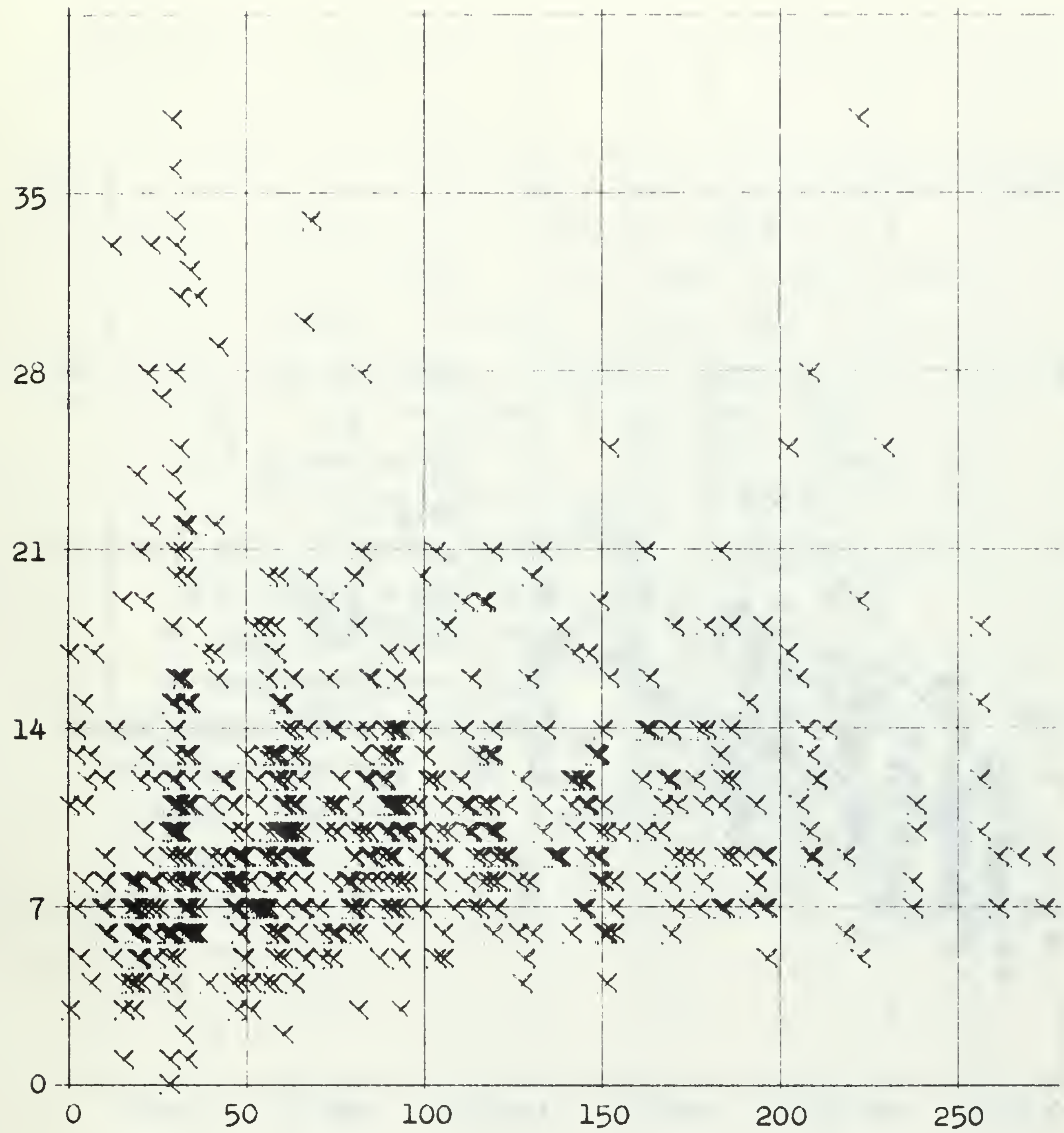
Y-SCALE = 4 UNITS PER INCH

HOURS SINCE OIL CHANGE

VS

PPM OF CHROMIUM

FIGURE 1



X-SCALE = 50 UNITS PER INCH

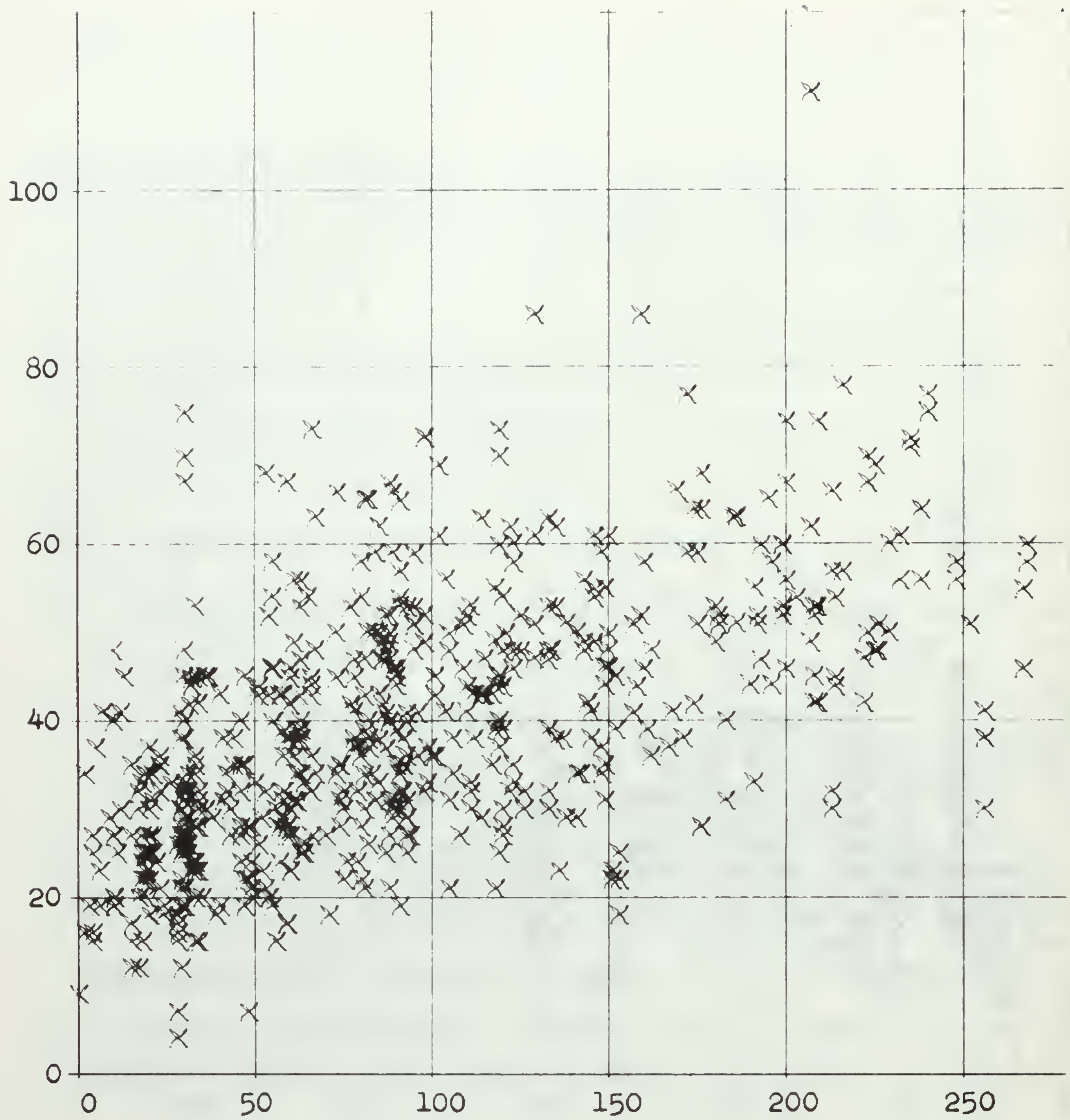
Y-SCALE = 7 UNITS PER INCH

HOURS SINCE OIL CHANGE

VS

PPM OF ALUMINUM

FIGURE 2



X-SCALE = 50 UNITS PER INCH

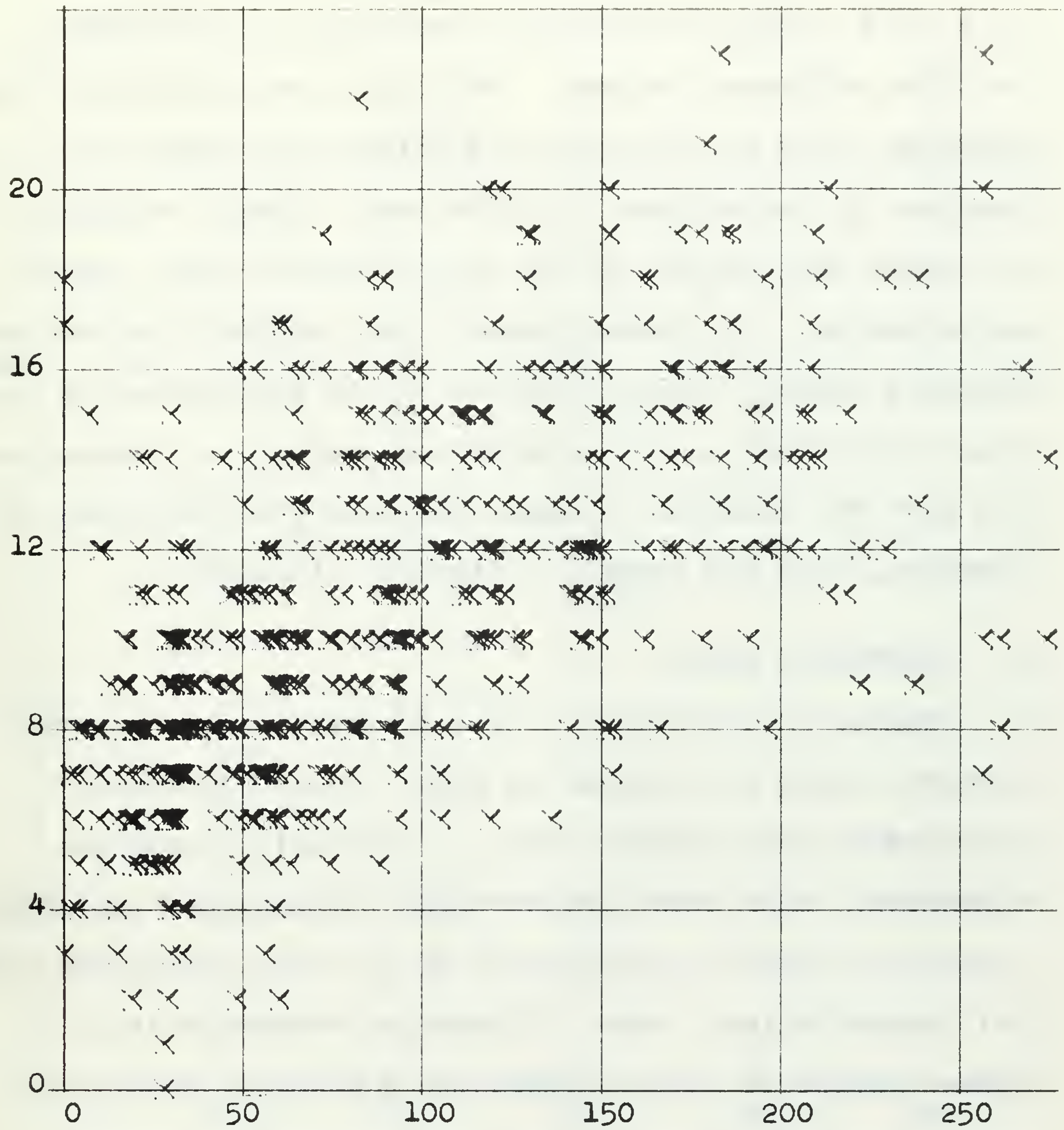
Y-SCALE = 20 UNITS PER INCH

HOURS SINCE OIL CHANGE

VS

PPM OF IRON

FIGURE 3



X-SCALE = 50 UNITS PER INCH

Y-SCALE = 4 UNITS PER INCH

HOURS SINCE OIL CHANGE

VS

PPM OF COPPER

FIGURE 4

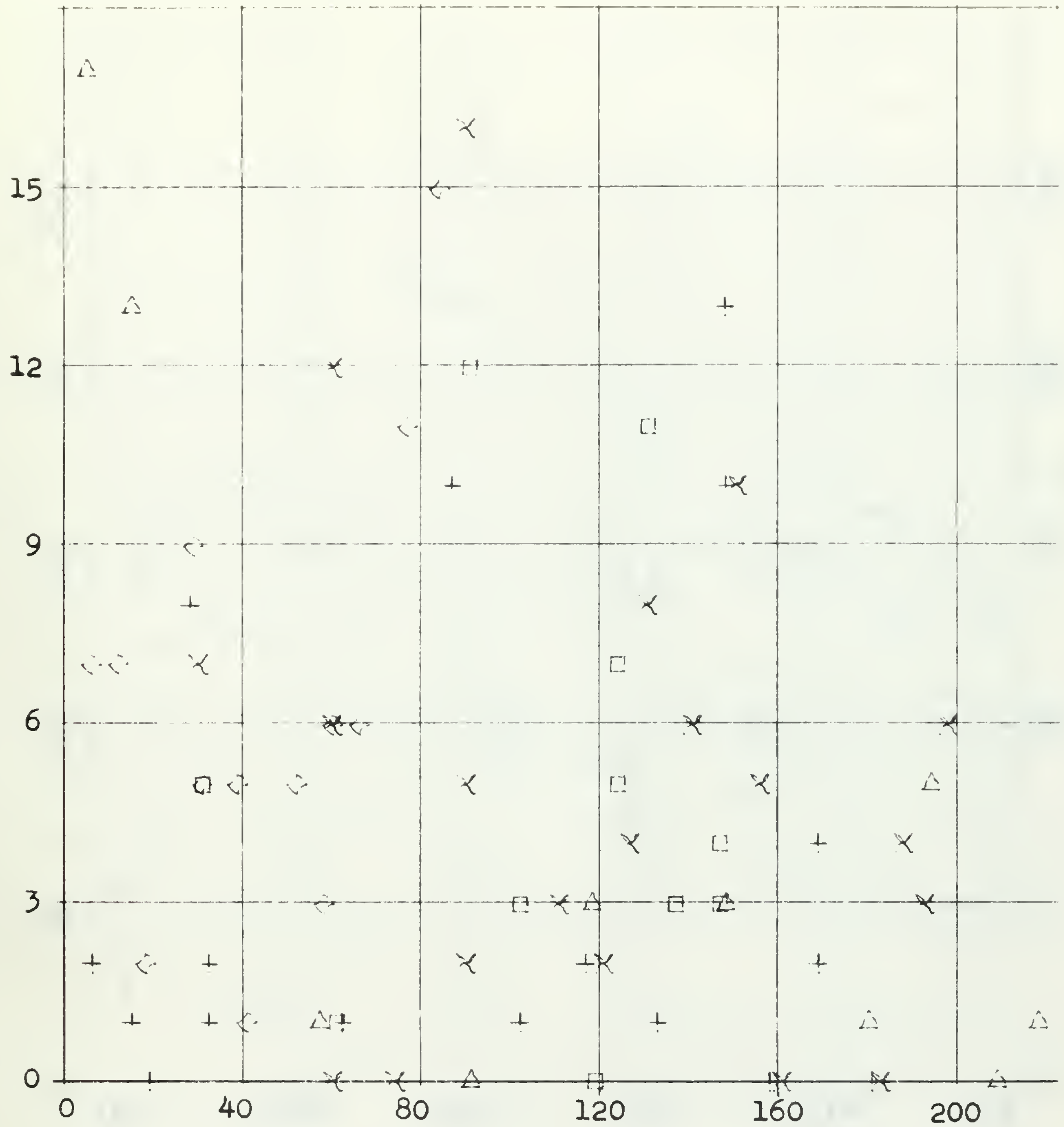
×, +, Δ, ◇, and □, were used on each plot to represent the five different engines. Thus, both the behavior of the readings for a given engine and differences among the readings of the engines could be seen. Again, evidence of trends was limited to the three elements iron, copper and aluminum. For these elements each of the five engines showed a roughly linear increase in the ppm content as the hours since the last oil change increased. For comparison, the plot for chromium is again included with the plots of aluminum, iron and copper in Figures 5 through 8.

C. REGRESSION MODEL

Because of the evidence of a linear increase in ppm content versus an increase in hours since oil change, provided by the computer plots, a regression model was suggested. With this type of model the expected content of a metallic element would change as the hours since the last oil change varies. Thus, differences between what is a normal amount of contamination for a properly operating engine just after its oil has been changed and several flying hours later could automatically be incorporated into a classification criteria.

1. Data Selection

Of all the engines of the R1820-82 model type represented on the tape, those with eight or more different analyses were selected. Of these, any with missing data on one or more records, which brought the usable number of



X-SCALE = 40 UNITS PER INCH

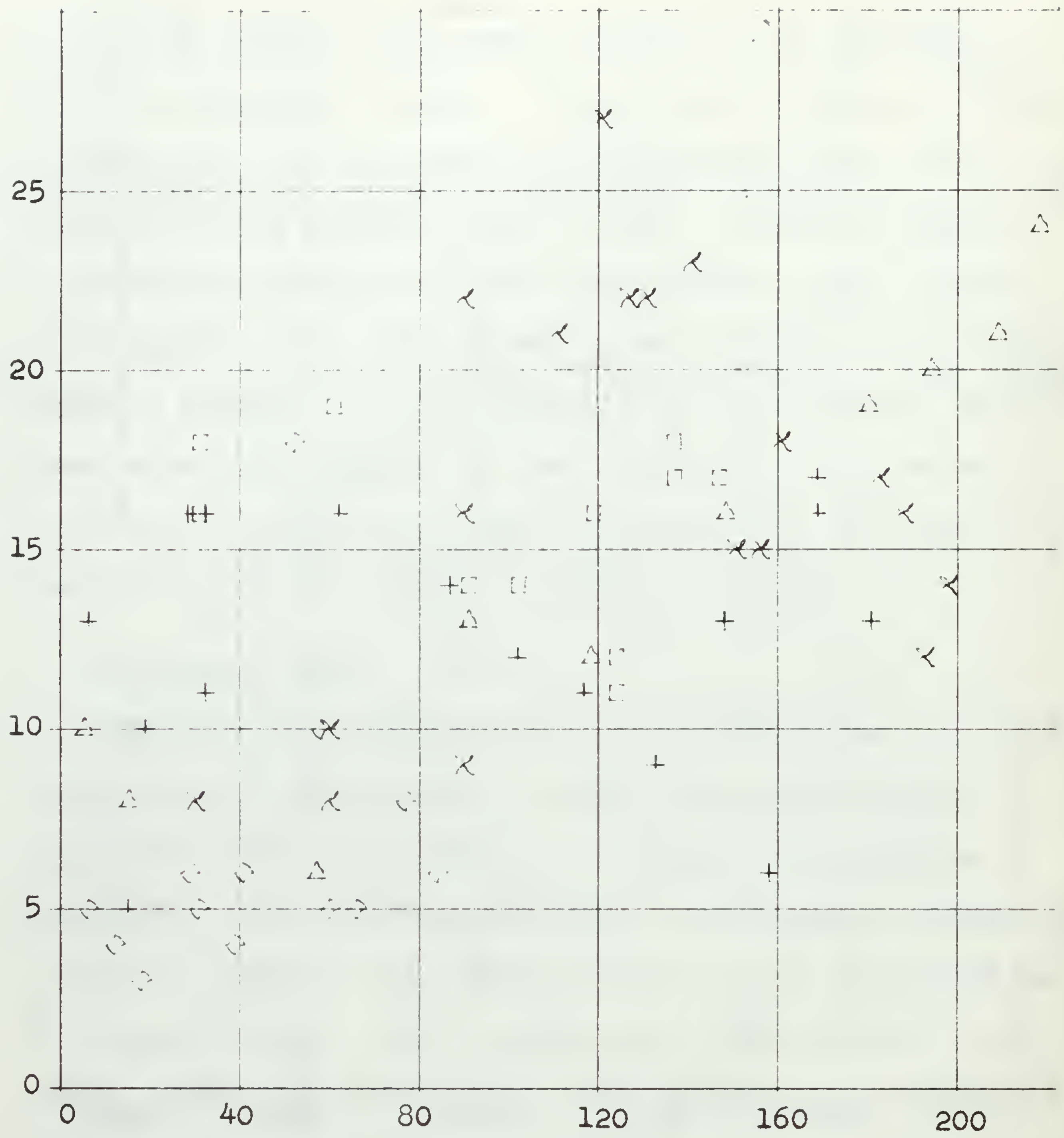
Y-SCALE = 3 UNITS PER INCH

HOURS SINCE OIL CHANGE

VS

PPM OF CHROMIUM

FIGURE 5



X-SCALE = 40 UNITS PER INCH

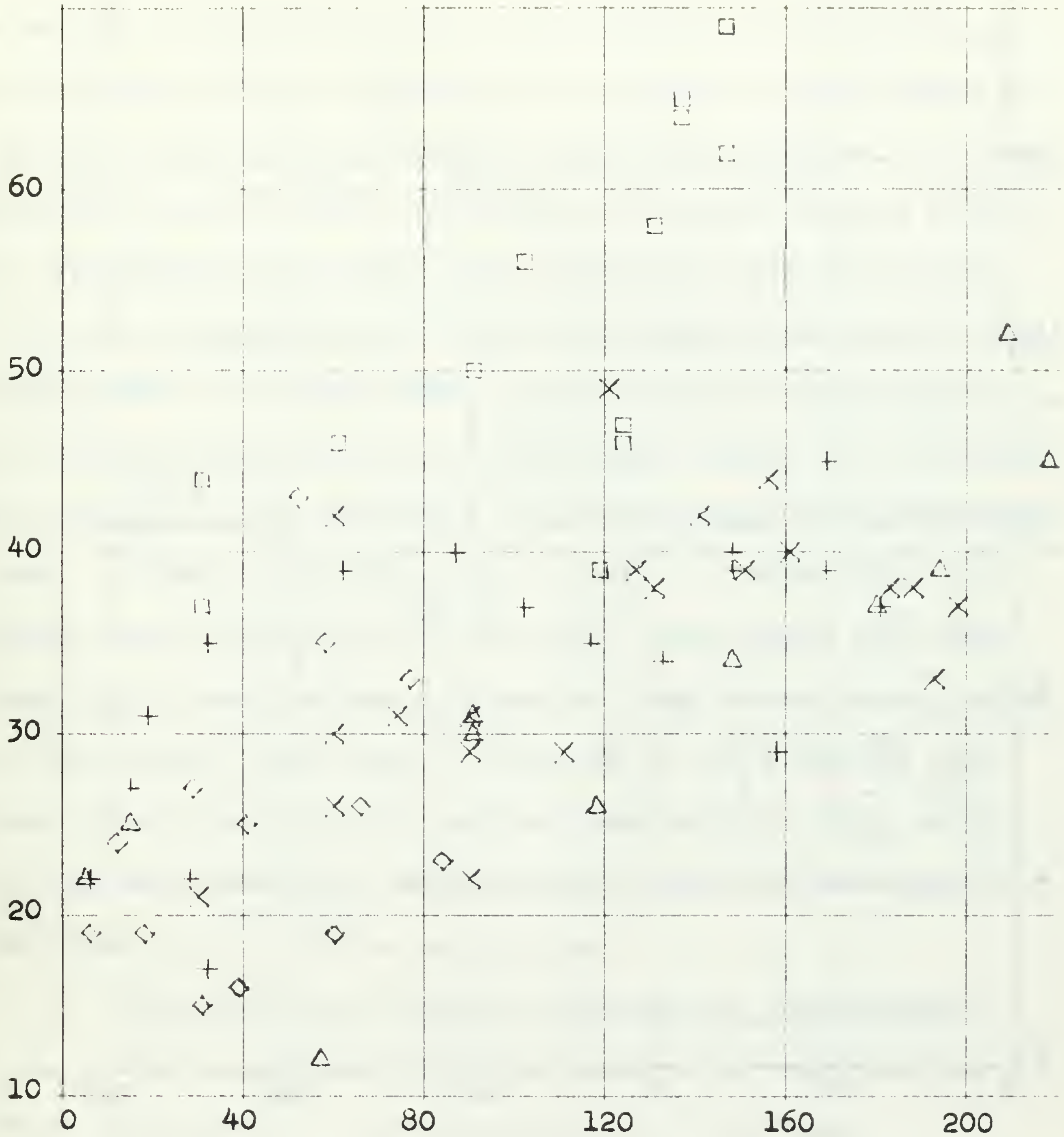
Y-SCALE = 5 UNITS PER INCH

HOURS SINCE OIL CHANGE

VS

PPM OF ALUMINUM

FIGURE 6



X-SCALE = 40 UNITS PER INCH

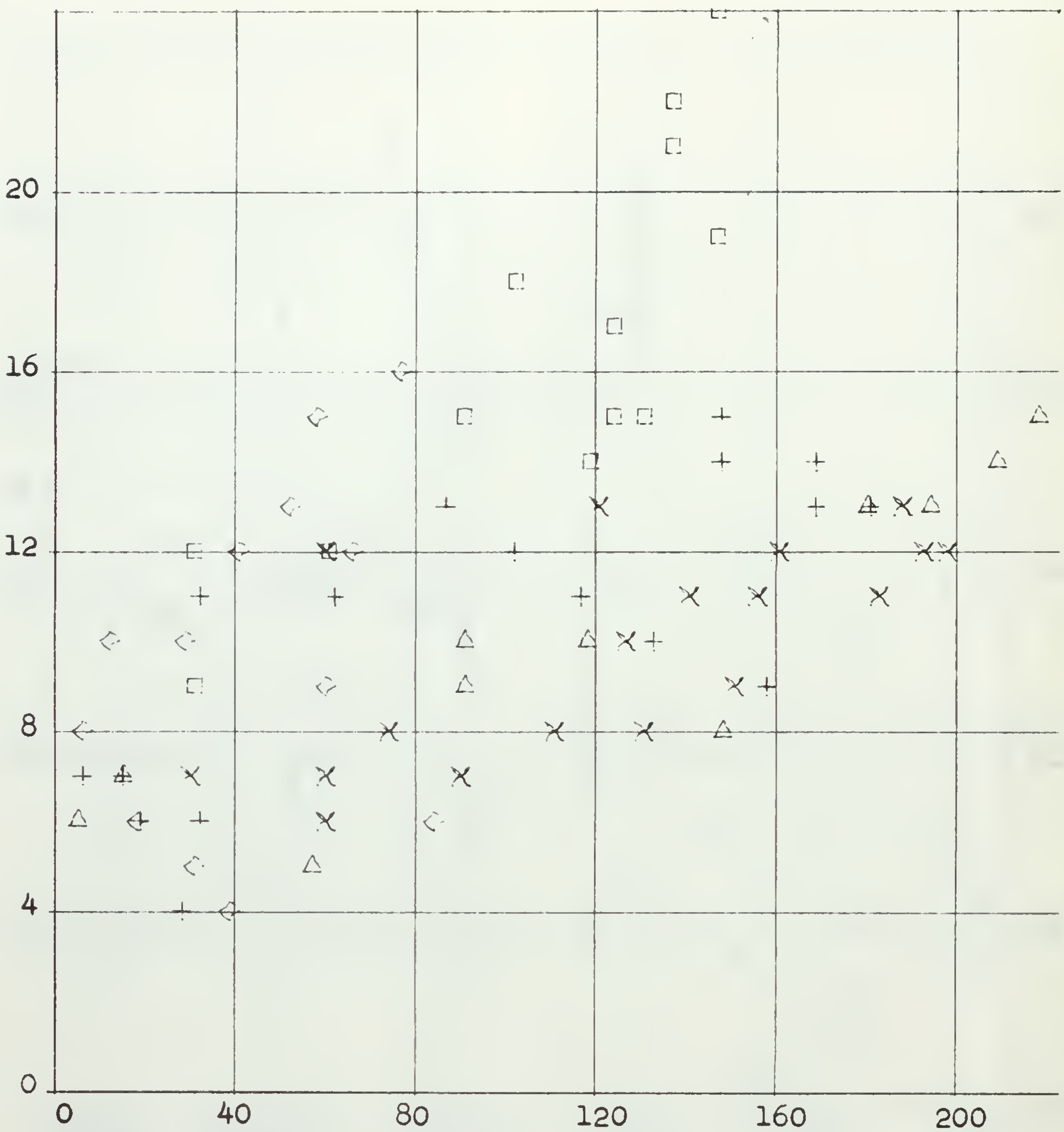
Y-SCALE = 10 UNITS PER INCH

HOURS SINCE OIL CHANGE

VS

PPM OF IRON

FIGURE 7



X-SCALE = 40 UNITS PER INCH

Y-SCALE = 4 UNITS PER INCH

HOURS SINCE OIL CHANGE

VS

PPM OF COPPER

FIGURE 8

readings to less than eight, was rejected. The remaining engines were then screened in an attempt to insure that all the data had come from properly operating engines. As was mentioned earlier, the tape did not provide data on either the recommendation made by the laboratory or the results of any maintenance which might have been recommended. For this reason, it seemed that the best way to insure that none of the data had come from a discrepant engine was to exclude all readings which exceeded the threshold limits presently used. These limits are given in Ref. 1. After this procedure had been applied to the data, any engine with less than eight readings was eliminated from further examination. At this point there were 27 engines of the R1820-82 type remaining, each of which was represented with from eight to fourteen readings. This was the data that was used in the remainder of the investigation.

2. Estimation and Tests of Regression Coefficients

It was assumed that the outcome of each spectrometric analysis on a given engine is of the form

$$\underline{Y} = \underline{BX} + \underline{e}$$

where $\underline{Y}$ is a seven-component vector of ppm metallic contents; $\underline{B}$ is a 7x2 matrix of unknown coefficients; $\underline{X}$ is a two-component vector with first component identically one and second component equal to the operating hours since the last oil change; and $\underline{e}$ is a multivariate normal error vector with mean vector zero and unknown covariance matrix,

$\underline{\Sigma}$. It will be recalled that the examination of the Air Force data in section III indicated that the covariance matrix may vary as the mean content vector changes. However, the slopes of the regression lines mentioned in part C of that section are such that variations in the mean vectors of the extent present in the operational data do not result in an appreciable variation in the elements of $\underline{\Sigma}$. For this reason, it will be assumed that the covariance matrix, $\underline{\Sigma}$, is constant in the development to follow. Under this assumption, the matrix $\underline{B}$ associated with each engine was estimated using the least-squares estimation technique described in Appendix B.

Since the random error vector is assumed to be $N(\underline{0}, \underline{\Sigma})$, each observation from a particular engine is $N(\underline{BX}, \underline{\Sigma})$. If the matrix $\underline{B}$ is partitioned so that

$$\underline{B} = (\underline{B}_1, \underline{B}_2),$$

then a test of the value of each of the components of $\underline{B}_2$ can be made to determine whether the variability of the readings for a specific element is related to variations in the hours since oil change. The details of this test are included in Appendix B. For each engine and each component of $\underline{B}_2$ a test of the hypothesis that the component is equal to zero was made. Table 1 gives the number of times a specific component was significantly positive or negative at an over-all α-level of .10. For this α value, the expected number of times in 27 tests the results will

be significantly positive or negative, if the component is actually zero, is 1.35.

TABLE 1

NUMBER OF SIGNIFICANT REGRESSION SLOPES
TWO-TAILED TEST; α = .10

$\underline{B}_2$	Element	Number of significantly positive components	Number of significantly negative components
$B_{2,1}$	Aluminum	8	0
$B_{2,2}$	Iron	17	0
$B_{2,3}$	Chromium	3	1
$B_{2,4}$	Silver	1	3
$B_{2,5}$	Copper	16	1
$B_{2,6}$	Magnesium	5	0
$B_{2,7}$	Nickel	5	0

These results indicate that the metallic content of aluminum, iron and copper in properly operating engines of the R1820-82 type tends to increase as the hours since the engine's oil was last changed increase. The evidence pointing to this conclusion is particularly strong in the case of iron and copper. Further, there seems to be no significant indication that such a relationship exists in general for chromium, silver, magnesium or nickel. The numerical results of the tests of the components of $\underline{B}_2$ for

each engine, in addition to the raw data and estimates of $\underline{B}$ and $\underline{\Sigma}$, are included in the Computer Output section.

Any use of these results in establishing an operational classification procedure for identifying discrepant engines depends upon the estimation of the unknown matrix, $\underline{B}$, from past analyses. For this reason, it is important to determine if the observations from different engines all come from the same over-all probability distribution. If this is the case, all data on engines of the R1820-82 model type could be used to estimate a single matrix, $\underline{B}$. As a first step in this direction, the model

$$\underline{Y}_i = \underline{B}_i \underline{X} + \underline{e}_i$$

was used where $\underline{Y}_i$ is a seven-component vector of readings on engine i; $\underline{B}_i = (\underline{B}_1, \underline{B}_2)_i$, a 7x2 matrix of coefficients associated with the $i^{\underline{th}}$ engine and where the components of $\underline{B}_2$ associated with chromium, silver, magnesium and nickel are assumed to be zero; and $\underline{e}_i$ is $N(\underline{0}, \underline{\Sigma}_i)$. The elements of $\underline{B}_i$, not assumed to be zero, were estimated as before and used to estimate the 27 covariance matrices, $\underline{\Sigma}_i$. The unbiased estimate of $\underline{\Sigma}_i$ is

$$\hat{\underline{\Sigma}}_i = \frac{1}{n_i - 2} \sum_{j=1}^{n_i} (\underline{Y}_{i,j} - \hat{\underline{B}}_i \underline{X}_j)(\underline{Y}_{i,j} - \hat{\underline{B}}_i \underline{X}_j)'$$

where $\underline{Y}_{i,j}$ is the $j^{\underline{th}}$ observation of the vector $\underline{Y}_i$; $\hat{\underline{B}}_i$ is the estimate of $\underline{B}_i$; $\underline{X}_j$ is the $j^{\underline{th}}$ observation of $\underline{X}$; and n_i is the number of observations associated with the $i^{\underline{th}}$

engine. A test of the hypothesis of equal covariance matrices was made using these estimates. The test statistic was extremely significant at the .10 level, and the hypothesis of equal covariance matrices was rejected. The test used and numerical results are included in Appendix A.

Since the evidence indicates that the covariance matrices associated with readings from different engines are not the same, the overall conjecture of like distributions must also be rejected.

V. CONCLUSION

The investigation of the Air Force data and the actual analysis records of a three month period from the Pensacola laboratory lead to three main conclusions. First, the error inherent in the ppm spectrometric readings is multivariate normally distributed with significant covariances existing between the readings of various pairs of elements. Further, there appears to be a linear increase in the content of aluminum, copper and iron present in properly operating engines of the R1820-82 type as the hours since the last oil change increase. Finally, there seems to be no justification for expecting readings on samples from different engines of the R1820-82 type to vary in the same manner. Based on these results, an objective classification criterion can be formulated which may be of use in imporving the present classification procedure.

For example, all back data on a particular engine of the R1820-82 type which was in proper working order could be used to estimate the matrix $\underline{B}$, which in turn could be used to estimate the covariance matrix $\underline{\Sigma}$. Then, any observation, $\underline{Y}$, of the spectrometric analysis of a new oil sample from that engine is distributed as $N(\underline{BX}, \underline{\Sigma})$ if the engine is operating properly. The estimates of $\underline{B}$ and $\underline{\Sigma}$ could be used to construct a confidence region $R_\alpha(x)$ [Appendix B]. The region $R_\alpha(x)$ would be constructed so that, if the engine is operating properly, the reading $\underline{Y}$

will be contained in the region $R_\alpha(x)$ with probability $1-\alpha$. Thus, one classification criterion would be: classify the engine as operating properly if $\underline{Y}$ is within $R_\alpha(x)$ and classify as discrepant otherwise. By making α small, say .01, the number of operational engines, which are mistakenly classified as discrepant, can be expected to be of the order of 1 in 100. However, the smaller the parameter α is made, the larger the region $R_\alpha(x)$ becomes, and thus, the more likely it is that a discrepant engine will be classified as operating properly.

For this reason, it may be more appropriate to use two values of α. For example, α could be set at .10 and α' at .01 and two regions $R_\alpha(x)$ and $R_{\alpha'}(x)$ constructed. In this way a procedure could be used which would 1) classify the engine as in proper working order if $\underline{Y}$ is in $R_\alpha(x)$; 2) classify as discrepant if $\underline{Y}$ is not in $R_{\alpha'}(x)$; and 3) require verification of $\underline{Y}$ or more frequent sampling if $\underline{Y}$ is in $R_{\alpha'}(x)$ but not in $R_\alpha(x)$.

The final selection of a specific classification criterion and the setting of the appropriate level(s) of α must be done subjectively and should be based upon an examination of the costs involved. If the cost of classifying a discrepant engine as operational is much larger than the cost of grounding an operational aircraft then α should be made appropriately large compared to its value if the reverse were true.

APPENDIX A

STATISTICAL TESTS

A. TRANSFORMATION OF $N(\underline{\mu}, \underline{\Sigma})$ TO $N(\underline{0}, \underline{I})$

If $\underline{X}$ is a multivariate normal random variable with mean vector, $\underline{\mu}$, and covariance matrix, $\underline{\Sigma}$ then

$$\underline{Z} = \underline{P}(\underline{X} - \underline{\mu})$$

is multivariate normally distributed with mean vector, $\underline{0}$, and covariance matrix, $\underline{P\Sigma P}'$ [Ref. 8]. In addition, since $\underline{\Sigma}$ is the symmetric matrix of a positive definite quadratic form, there exists an orthogonal matrix, $\underline{B}$, such that

$$\underline{B\Sigma B}' = \underline{D}$$

where $\underline{D}$ is a diagonal matrix with all diagonal elements positive [Ref. 9]. The matrix, $\underline{B}$, can be constructed using the characteristic vectors of $\underline{\Sigma}$ as columns of $\underline{B}$. Further, if the matrix $\underline{C}$ is defined as the diagonal matrix which has diagonal elements equal to the inverse of the square-root of the corresponding element of $\underline{D}$, then

$$\underline{CB\Sigma B}'\underline{C}' = \underline{CDC}' = \underline{I}$$

where $\underline{I}$ is the identity matrix. Thus, if we define

$$\underline{P} = \underline{CB}$$

then

$$\underline{Z} = \underline{P}(\underline{Y} - \underline{\mu})$$

is multivariate normal with zero mean vector and identity covariance matrix, and the elements of the vector $\underline{Z}$ are mutually stochastically independent standard normal random variables.

B. KOLMOGOROV-SMIRNOV TEST OF GOODNESS OF FIT

Let F(x) be defined as the cumulative distribution function of the random variable X which is N(0,1). In addition, define $S_n(x)$ to be the sample cumulative distribution function based on a set of n observations of a random variable assumed to be N(0,1). Then

$$S_n(x) = k/n$$

where k is the number of observations in the sample which are less than or equal to x. Then, the Kolmogorov-Smirnov test statistic D is defined as

$$D = \max_x |F(x) - S_n(x)|.$$

Observed values of D can be compared with its tabled distribution to determine the acceptability of the normal hypothesis [Ref. 10].

C. TEST FOR EQUALITY OF SEVERAL COVARIANCE MATRICES

Suppose $\underline{Y}_1$, $\underline{Y}_2, \ldots, \underline{Y}_q$ are p-component multivariate normal random variables with distribution denoted $N(\underline{\mu}_i, \underline{\Sigma}_i)$, $i = 1,2,\ldots,q$. In order to test the hypothesis

$$\underline{\Sigma}_i = \underline{\Sigma}_j \text{ for all } i,j = 1,2,\ldots,q,$$

based on q samples of size N_i from the distribution of Y_i, $i = 1,2,\ldots,q$, let the following quantities be defined:

$$n_i = N_i - 1, \quad i = 1,2,\ldots,q;$$

$$n = \sum_{i=1}^{q} n_i;$$

$$\underline{A}_i = \sum_{k=1}^{n_i} (\underline{Y}_{i,k} - \overline{\underline{Y}}_i)(\underline{Y}_{i,k} - \overline{\underline{Y}}_i)' \quad i = 1,2,\ldots,q;$$

and

$$A = \sum_{i=1}^{q} \underline{A}_i.$$

Then the test statistic is

$$W = k[\sum_{i=1}^{q} n_i(\log|\hat{\underline{\Sigma}}| - \log|\hat{\underline{\Sigma}}_i|)]$$

where

$$k = 1 - [\sum_{i=1}^{q} \frac{1}{n_i} - \frac{1}{n}] \frac{2p^2 + 3p - 1}{6(p+1)(q-1)},$$

$$\hat{\underline{\Sigma}} = \underline{A}/n$$

and

$$\hat{\underline{\Sigma}}_i = \underline{A}_i/n_i \quad i = 1,2,\ldots,q.$$

Asymptotic expansion of the distribution of this test statistic results in a distribution described by the following probability statement for W,

$$\Pr(W \le w) = \Pr(\chi^2_f \le w) - c[\Pr(\chi^2_{f+4} \le w) - \Pr(\chi^2_f \le w)] - 0(n^{-3})$$

where

$$c = \frac{p(p+1)[(p-1)(p+2)(\sum_{i=1}^{q} \frac{1}{n_i^2} - \frac{1}{n^2}) - 6(q-1)(1-k)^2]}{48k^2}$$

$$f = \tfrac{1}{2}(q-1)(p+1)p$$

and χ^2_f denotes a chi-square random variable with f degrees of freedom [Ref. 8].

This test was applied to the Air Force data as described in III C, with the resulting values of W = 478.5, f = 252 and c = 5.75. The test statistic is extremely significant at the .01 level and the hypothesis was rejected.

In addition, the test was used on the operational data as described in IV C 2 where now

$$\underline{\hat{\Sigma}} = A/(n-q)$$

and

$$\underline{\hat{\Sigma}}_i = A_i/(n_i-1) \qquad i = 1,2,\ldots,q$$

instead of as defined above. The results for this test were W = 936.2, f = 728 and c = 14.9. Once again the test statistic is extremely significant at the .01 level and the hypothesis was rejected.

D. TEST FOR INDEPENDENCE AMONG A SET OF NORMAL VARIATES

If the p-component vector $\underline{Y}$ has a multivariate normal distribution described by $N(\underline{\mu}, \underline{\Sigma})$, then the test of zero covariance,

$$E[(y_i - \mu_i)(y_j - \mu_j)] = 0,$$

for all $i \neq j$ and $i,j = 1,2,\ldots,p$ is equivalent to a test of the stochastic independence of the components of $\underline{Y}$. Suppose a sample, $\underline{Y}_1, \underline{Y}_2, \ldots, \underline{Y}_n$, of n observations from the distribution of $\underline{Y}$ is obtained, then let V equal the determinant of the correlation matrix, R, defined by equation (2). The asymptotic expansion of the distribution of V results in the probability statement [Ref. 8],

$$\Pr(-m\log V \leq v) = \Pr(\chi_f^2 \leq v) + \frac{c}{m^2}[\Pr(\chi_{f+4}^2 \leq v) - \Pr(\chi_f^2 \leq v)] + 0(m^{-3})$$

where

$$m = n - \frac{2p + 11}{6},$$

$$f = \tfrac{1}{2}p(p-1)$$

and

$$c = \frac{p(p-1)(2p^2 - 2p - 13)}{288}.$$

This test was applied to the data for the Pensacola laboratory accumulated in the Air Force experiment as described in section III D. Table 2 gives the numerical

results for the ten groups of samples tested and the test outcomes for a level of significance of .10.

TABLE 2

RESULTS OF TEST FOR INDEPENDENCE

$f = 21$ $m = 5.833$

$c = 10.35$ $\alpha = .10$

Sample Group	Test Statistic	Result
1	26.20	Accept
2	39.10	Reject
3	30.81	Reject
4	31.57	Reject
5	38.07	Reject
6	36.36	Reject
7	23.78	Accept
8	41.60	Reject
9	23.63	Accept
10	35.17	Reject

APPENDIX B

REGRESSION: ESTIMATION, TESTS AND PREDICTION

A. ESTIMATION OF REGRESSION PARAMETERS

Let $\underline{Y}$ be a seven-component random vector with distribution $N(\underline{BX}, \underline{\Sigma})$, where $\underline{B}$ is an unknown 7x2 matrix of coefficients, $\underline{X}$ is of the form $(1,x)'$, and $\underline{\Sigma}$ is an unknown covariance matrix which is constant for all values of $\underline{X}$. Then, the unbiased estimate of $\underline{B}$ [Ref. 8], based on a sample of size n with the $i^{\underline{th}}$ from $N(\underline{BX}_i, \underline{\Sigma})$ is

$$\hat{\underline{B}} = \underline{CA}^{-1}$$

where

$$\underline{C} = \sum_{i=1}^{n} \underline{Y}_i \underline{X}_i'$$

and

$$A = \sum_{i=1}^{n} \underline{X}_i \underline{X}_i' \ .$$

This estimate is normally distributed with mean matrix, B, and covariance matrix $\underline{\Sigma} \otimes A^{-1}$, where the symbol, $\otimes$, denotes the Kronecker product [Ref. 11]. Further, the unbiased estimate of $\underline{\Sigma}$ [Ref. 8] is

$$\hat{\underline{\Sigma}} = \frac{1}{n-2} \sum_{i=1}^{n} (\underline{Y}_i - \hat{\underline{B}}\underline{X}_i)(\underline{Y}_i - \hat{\underline{B}}\underline{X}_i)'$$

and $(n-2)\hat{\underline{\Sigma}}$ has a Wishart distribution with parameters $\underline{\Sigma}$ and n-2.

B. TEST OF THE VALUE OF REGRESSION COEFFICIENTS

Suppose the 7x2 matrix $\underline{B}$ is partitioned in the form

$$B = (\underline{B}_1, \underline{B}_2) .$$

Then for any non-null vector of constants, $\underline{C}$, the hypothesis

$$\underline{C}'\underline{B}_2 = \underline{C}'\underline{B}^*$$

can be tested using an F statistic [Ref. 12]. By choosing $\underline{C}$ to be the vector with 1 as the $i^{\underline{th}}$ component and all other components zero, and the vector $\underline{B}^*$ to be the null vector, the hypotheses

$$B_{2i} = 0 \qquad i = 1,2,\ldots,7$$

can be tested. With this selection, the standard t-test of the slope of a regression line [Ref. 6 and 7] can be used. In this way, each of the components of $\underline{B}_2$ can be tested individually, each with an assigned level of significance, α.

C. CONSTRUCTION OF THE REGION, $R_\alpha(x)$

If the matrices $\underline{\hat{B}}$ and $\underline{\hat{\Sigma}}$ are estimated with a sample of size n in the manner described above, and if $\underline{Y}$ is a new observation from $N(\underline{BX}^*, \underline{\Sigma})$, the confidence region $R_\alpha(x)$ can be constructed so that $\underline{Y}$ will be in $R_\alpha(x)$ with probability at least $1-\alpha$. The estimate of the mean of $\underline{Y}$ is

$$\underline{\hat{Y}} = \underline{\hat{B}X}^*$$

and has covariance matrix,

$$\underline{S} = \underline{T}(\underline{A} \otimes \hat{\underline{\Sigma}}^{-1})^{-1}\underline{T}'$$

where

$$\underline{A} = \sum_{i=1}^{n} \underline{X}_i\underline{X}_i'$$

and

$$\underline{T} = X^* \otimes I_7.$$

Thus,

$$(\hat{\underline{Y}} - \underline{BX}^*)' \underline{S}^{-1}(\hat{\underline{Y}} - \underline{BX}^*)$$

has Hotelling's T^2 distribution [Refs. 8 and 5]. Hence, the set of vectors $\underline{m}$ satisfying

$$(\hat{\underline{Y}} - \underline{m})' \underline{S}^{-1}(\hat{\underline{Y}} - \underline{m}) \leq T^2(\alpha)$$

comprise a 100 $(1-\alpha)$% confidence region for $\underline{BX}^*$ [Refs. 8 and 5]. Since the region, $R_\alpha(x)$, is to place bounds on $\underline{Y}$, which has covariance matrix, $\underline{\Sigma}$, it can be defined by

$$(\hat{\underline{Y}} - \underline{m})'(\underline{S} + \hat{\underline{\Sigma}})^{-1}(\hat{\underline{Y}} - \underline{m}) \leq T^2(\alpha).$$

The set of all vectors, $\underline{m}$, satisfying this constraint form a confidence region, $R_\alpha(x)$, which will contain $\underline{Y}$ with probability at least $1 - \alpha$ if $\underline{Y}$ came from $N(\underline{BX}^*, \underline{\Sigma})$.

COMPUTER OUTPUT

A. DATA FROM AIRFORCE EXPERIMENT

GROUP # 1

AL	FE	CR	AG	CU	MG	NI
3	76	12	4	62	3	5
1	72	8	3	58	3	1
0	79	6	2	63	3	1
1	79	10	4	61	3	5
1	73	4	2	61	2	1
0	75	9	4	61	3	1
3	73	11	4	59	3	2
1	79	3	2	66	3	1
0	74	2	3	62	3	0

GROUP # 2

AL	FE	CR	AG	CU	MG	NI
0	127	7	1	12	10	2
0	142	10	2	13	11	3
1	144	15	3	14	11	3
0	147	9	2	14	11	3
0	143	9	3	14	11	3
0	154	14	3	15	12	4
0	145	17	4	15	11	4
0	153	14	3	15	13	4
2	77	11	5	6	14	1
0	84	8	4	8	16	2

GROUP # 3

AL	FE	CR	AG	CU	MG	NI
0	151	14	3	16	12	3
1	151	13	2	14	11	4
1	84	9	4	8	16	2
0	84	12	4	6	15	1
0	80	10	4	5	14	2
0	82	7	4	7	15	1
0	83	11	5	8	16	1
0	88	6	4	7	17	0
1	77	7	4	6	14	1
1	82	8	3	8	15	1

GROUP # 4

AL	FE	CR	AG	CU	MG	NI
26	27	9	4	35	10	13
23	27	8	4	36	11	13
24	27	15	4	37	10	14
20	28	11	4	34	11	13
24	28	15	5	34	10	14
23	29	12	5	34	11	14
22	28	10	5	34	10	13
23	28	14	5	35	10	14

GROUP # 5

AL	FE	CR	AG	CU	MG	NI
2	137	9	4	100	6	1
5	132	17	6	107	5	1
6	127	17	6	103	5	1
3	133	2	4	116	5	1
4	141	4	3	111	6	0
3	138	6	4	117	6	1
2	139	12	5	103	5	1
4	145	18	6	114	6	2
3	137	11	5	109	5	1
3	133	10	5	104	6	1

GROUP # 6

AL	FE	CR	AG	CU	MG	NI
0	112	11	2	13	19	4
0	105	5	2	11	19	2
1	107	9	3	13	19	2
2	98	10	4	13	17	3
0	109	8	2	12	18	2
0	104	11	3	15	19	3
0	110	10	3	15	20	4
0	115	11	3	15	21	3
1	110	13	3	14	20	3
1	125	11	3	16	22	3

GROUP # 7

AL	FE	CR	AG	CU	MG	NI
3	99	4	4	88	27	1
4	95	8	4	79	25	1
1	87	10	2	72	25	2
3	102	11	4	86	27	1
4	104	10	4	90	28	1
3	107	10	5	79	27	2
3	105	11	5	83	28	5
6	106	11	4	87	28	1
1	104	8	4	81	27	1

GROUP # 8

AL	FE	CR	AG	CU	MG	NI
2	132	6	3	5	11	1
0	122	8	5	5	11	3
0	127	4	3	8	11	3
0	133	11	4	7	12	4
1	140	4	2	6	12	2
1	138	1	2	6	13	1
0	127	2	3	6	11	2
0	130	2	2	8	12	2
0	137	7	4	7	12	3
0	128	9	4	8	11	4

GROUP # 9

AL	FE	CR	AG	CU	MG	NI
2	17	12	3	9	12	1
1	18	11	2	10	12	0
3	17	11	2	10	13	1
0	17	8	1	10	12	0
1	17	13	2	10	12	0
1	19	10	2	11	13	1
1	18	6	1	11	12	1
1	18	9	2	11	12	1

GROUP # 10

AL	FE	CR	AG	CU	MG	NI
0	187	8	3	7	10	4
1	188	10	3	8	10	7
1	197	12	3	9	9	4
0	190	8	3	7	10	4
0	0	0	0	0	0	0
0	188	8	2	9	10	4
1	212	7	2	10	10	3
0	206	5	1	10	11	4
0	217	8	2	10	11	5

B. SAMPLE MEANS AND STANDARD DEVIATIONS

GROUP # 1:

	AL	FE	CR	AG	CU	MG	NI
MEAN	1.11	75.56	7.22	3.11	61.44	2.89	1.89
S.D.	1.167	2.835	3.632	0.928	2.299	0.333	1.833

GROUP # 2:

	AL	FE	CR	AG	CU	MG	NI
MEAN	0.20	145.70	12.20	2.60	14.20	11.30	3.30
S.D.	0.422	7.848	3.225	0.843	1.135	0.823	0.675

GROUP # 3:

	AL	FE	CR	AG	CU	MG	NI
MEAN	0.50	82.10	8.90	4.10	6.90	15.20	1.20
S.D.	0.707	3.384	2.025	0.568	1.101	1.033	0.632

GROUP # 4:

	AL	FE	CR	AG	CU	MG	NI
MEAN	23.12	27.75	11.75	4.50	34.88	10.37	13.50
S.D.	1.727	0.707	2.712	0.535	1.126	0.518	0.535

GROUP # 5:

	AL	FE	CR	AG	CU	MG	NI
MEAN	3.50	136.20	10.60	4.80	108.40	5.50	1.00
S.D.	1.269	5.119	5.582	1.033	5.969	0.527	0.471

GROUP # 6:

	AL	FE	CR	AG	CU	MG	NI
MEAN	0.50	109.70	9.90	2.80	13.70	19.40	2.90
S.D.	0.707	7.530	2.183	0.632	1.567	1.430	0.738

GROUP # 7:

	AL	FE	CR	AG	CU	MG	NI
MEAN	3.11	100.22	9.22	4.00	82.78	26.89	1.67
S.D.	1.537	6.039	2.279	0.866	5.653	1.168	1.323

GROUP # 8:

	AL	FE	CR	AG	CU	MG	NI
MEAN	0.40	131.40	5.40	3.20	6.60	11.60	2.50
S.D.	0.699	5.702	3.340	1.033	1.174	0.699	1.080

GROUP # 9:

	AL	FE	CR	AG	CU	MG	NI
MEAN	1.25	17.62	10.00	1.87	10.25	12.25	0.63
S.D.	0.886	0.744	2.268	0.641	0.707	0.463	0.518

GROUP # 10:

	AL	FE	CR	AG	CU	MG	NI
MEAN	0.33	198.56	8.00	2.33	8.78	10.11	4.33
S.D.	0.500	11.294	2.062	0.707	1.202	0.601	1.118

C. SAMPLE CORRELATION MATRICES

GROUP # 1:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	-0.210	0.642	0.449	-0.254	0.036	0.591
FE	-0.210	1.000	-0.050	-0.169	0.746	0.338	0.350
CR	0.642	-0.050	1.000	0.808	-0.507	0.333	0.736
AG	0.449	-0.169	0.808	1.000	-0.495	0.449	0.596
CU	-0.254	0.746	-0.507	-0.495	1.000	0.073	-0.076
MG	0.036	0.338	0.333	0.449	0.073	1.000	0.182
NI	0.591	0.350	0.736	0.596	-0.076	0.182	1.000

GROUP # 2:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	0.121	0.294	-0.062	-0.093	-0.192	0.156
FE	0.121	1.000	0.631	0.568	0.831	0.807	0.816
CR	0.294	0.631	1.000	0.809	0.747	0.519	0.735
AG	-0.062	0.568	0.809	1.000	0.789	0.512	0.625
CU	-0.093	0.831	0.747	0.789	1.000	0.761	0.638
MG	-0.192	0.807	0.519	0.512	0.761	1.000	0.620
NI	0.156	0.816	0.735	0.625	0.638	0.620	1.000

GROUP # 3:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	-0.627	0.116	0.138	-0.071	-0.456	0.000
FE	-0.627	1.000	-0.225	-0.237	0.510	0.916	-0.218
CR	0.116	-0.225	1.000	0.493	-0.254	-0.308	0.278
AG	0.138	-0.237	0.493	1.000	-0.160	-0.038	-0.062
CU	-0.071	0.510	-0.254	-0.160	1.000	0.704	0.032
MG	-0.456	0.916	-0.308	-0.038	0.704	1.000	-0.238
NI	0.000	-0.218	0.278	-0.062	0.032	-0.238	1.000

GROUP # 4:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	-0.439	0.069	-0.077	0.377	-0.540	0.232
FE	-0.439	1.000	0.261	0.756	-0.763	0.293	0.378
CR	0.069	0.261	1.000	0.394	0.082	-0.433	0.887
AG	-0.077	0.756	0.394	1.000	-0.593	-0.258	0.500
CU	0.377	-0.763	0.082	-0.593	1.000	-0.153	0.119
MG	-0.540	0.293	-0.433	-0.258	-0.153	1.000	-0.258
NI	0.232	0.378	0.887	0.500	0.119	-0.258	1.000

GROUP # 5:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	-0.444	0.518	0.509	0.044	-0.249	0.000
FE	-0.444	1.000	-0.110	-0.286	0.369	0.536	0.184
CR	0.518	-0.110	1.000	0.929	-0.385	-0.227	0.591
AG	0.509	-0.286	0.929	1.000	-0.256	-0.408	0.685
CU	0.044	0.369	-0.385	-0.256	1.000	0.142	0.119
MG	-0.249	0.536	-0.227	-0.408	0.142	1.000	0.000
NI	0.000	0.184	0.591	0.685	0.119	0.000	1.000

GROUP # 6:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	-0.198	0.252	0.745	0.050	-0.220	-0.106
FE	-0.198	1.000	0.322	-0.200	0.575	0.890	0.214
CR	0.252	0.322	1.000	0.467	0.737	0.370	0.614
AG	0.745	-0.200	0.467	1.000	0.493	-0.024	0.191
CU	0.050	0.575	0.737	0.493	1.000	0.704	0.548
MG	-0.220	0.890	0.370	-0.024	0.704	1.000	0.253
NI	-0.106	0.214	0.614	0.191	0.548	0.253	1.000

GROUP # 7:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	0.469	0.206	0.376	0.593	0.426	-0.164
FE	0.469	1.000	0.205	0.741	0.764	0.909	0.089
CR	0.206	0.205	1.000	0.063	-0.122	0.246	0.359
AG	0.376	0.741	0.063	1.000	0.460	0.618	0.327
CU	0.593	0.764	-0.122	0.460	1.000	0.773	-0.228
MG	0.426	0.909	0.246	0.618	0.773	1.000	0.216
NI	-0.164	0.089	0.359	0.327	-0.228	0.216	1.000

GROUP # 8:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	0.457	-0.219	-0.431	-0.596	0.136	-0.736
FE	0.457	1.000	-0.196	-0.543	-0.040	0.770	-0.343
CR	-0.219	-0.196	1.000	0.812	0.045	-0.304	0.770
AG	-0.431	-0.543	0.812	1.000	-0.110	-0.492	0.697
CU	-0.596	-0.040	0.045	-0.110	1.000	0.054	0.526
MG	0.136	0.770	-0.304	-0.492	0.054	1.000	-0.294
NI	-0.736	-0.343	0.770	0.697	0.526	-0.294	1.000

GROUP # 9:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	-0.271	0.426	0.566	-0.342	0.522	0.545
FE	-0.271	1.000	-0.339	-0.112	0.747	0.311	0.325
CR	0.426	-0.339	1.000	0.786	-0.624	0.136	-0.243
AG	0.566	-0.112	0.786	1.000	-0.552	0.120	0.269
CU	-0.342	0.747	-0.624	-0.552	1.000	0.218	0.293
MG	0.522	0.311	0.136	0.120	0.218	1.000	0.447
NI	0.545	0.325	-0.243	0.269	0.293	0.447	1.000

GROUP # 10:

	AL	FE	CR	AG	CU	MG	NI
AL	1.000	0.030	0.606	0.354	0.139	-0.555	0.224
FE	0.030	1.000	-0.376	-0.621	0.830	0.506	-0.264
CR	0.606	-0.376	1.000	0.772	-0.303	-0.706	0.380
AG	0.354	-0.621	0.772	1.000	-0.784	-0.686	0.316
CU	0.139	0.830	-0.303	-0.784	1.000	0.385	-0.217
MG	-0.555	0.506	-0.706	-0.686	0.385	1.000	0.124
NI	0.224	-0.264	0.380	0.316	-0.217	0.124	1.000

D. RESULTS OF REGRESSION OF VARIANCE ON MEAN SQUARED

ALUMINUM

INTERCEPT:	0.8768E 00	RESIDUAL VARIANCE:	0.4118E 01
SLOPE:	0.4016E-02	VARIANCE OF INTERCEPT:	0.5782E-01
CORRELATION:	0.7067E 00	VARIANCE OF SLOPE:	0.1824E-05

IRON

INTERCEPT:	-0.4542E 01	RESIDUAL VARIANCE:	0.1829E 04
SLOPE:	0.3080E-02	VARIANCE OF INTERCEPT:	0.5537E 02
CORRELATION:	0.9296E 00	VARIANCE OF SLOPE:	0.8305E-07

CHROMIUM

INTERCEPT:	0.7276E 01	RESIDUAL VARIANCE:	0.5985E 03
SLOPE:	0.2642E-01	VARIANCE OF INTERCEPT:	0.5717E 02
CORRELATION:	0.1193E 00	VARIANCE OF SLOPE:	0.8870E-03

SILVER

INTERCEPT:	0.5494E 00	RESIDUAL VARIANCE:	0.7648E 00
SLOPE:	0.7357E-02	VARIANCE OF INTERCEPT:	0.4397E-01
CORRELATION:	0.1654E 00	VARIANCE OF SLOPE:	0.5800E-04

COPPER

INTERCEPT:	0.3822E 00	RESIDUAL VARIANCE:	0.1631E 03
SLOPE:	0.3238E-02	VARIANCE OF INTERCEPT:	0.2883E 01
CORRELATION:	0.9494E 00	VARIANCE OF SLOPE:	0.1054E-06

MAGNES.

INTERCEPT:	0.2444E 00	RESIDUAL VARIANCE:	0.1503E 01
SLOPE:	0.2224E-02	VARIANCE OF INTERCEPT:	0.3734E-01
CORRELATION:	0.7544E 00	VARIANCE OF SLOPE:	0.2511E-06

NICKEL

INTERCEPT:	0.1066E 01	RESIDUAL VARIANCE:	0.8284E 01
SLOPE:	-0.4177E-02	VARIANCE OF INTERCEPT:	0.1239E 00
CORRELATION:	-0.2370E 00	VARIANCE OF SLOPE:	0.3126E-04

RESULTS OF A T TEST OF THE HYPOTHESIS THAT THE SLOPE OF THE REGRESSION LINE IS EQUAL TO ZERO

ALUMINUM	0.2973542E 01	REJECT
IRON	0.1068822E 02	REJECT
CHROMIUM	0.8869581E 00	ACCEPT
SILVER	0.9660166E 00	ACCEPT
COPPER	0.9976842E 01	REJECT
MAGNES.	0.4439077E 01	REJECT
NICKEL	-0.7471411E 00	ACCEPT

E. OPERATIONAL DATA, ESTIMATED PARAMETERS AND TESTS OF REGRESSION SLOPES FOR ENGINES OF THE R1820-82 MODEL

RESULTS OF LINEAR REGRESSION USING THE MODEL,

$$Y = BX + E$$

WHERE:

Y IS A VECTOR OF WEAR METAL CONTAMINATION IN OIL SAMPLES

Y1 = PPM OF ALUMINUM
Y2 = PPM OF IRON
Y3 = PPM OF CHROMIUM
Y4 = PPM OF SILVER
Y5 = PPM OF COPPER
Y6 = PPM OF MAGNESIUM
Y7 = PPM OF NICKEL

B IS A 7X2 MATRIX OF COEFFICIENTS

X IS A VECTOR OF VARIABLES

X1 = 1
X2 = HOURS SINCE OIL CHANGE

AND E IS A 7 COMPONENT MULTIVARIATE NORMAL RANDOM ERROR VECTOR WITH MEAN VECTOR ZERO AND UNKNOWN COVARIANCE MATRIX SIGMA

ENGINE NUMBER: 316726[illegible]

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
6.0	25.0	6.0	2.0	12.0	3.0	1.0	11.0
8.0	28.0	8.0	2.0	11.0	1.0	2.0	57.0
10.0	41.0	3.0	3.0	23.0	2.0	2.0	256.0
9.0	25.0	1.0	2.0	12.0	1.0	1.0	21.0
9.0	31.0	6.0	2.0	10.0	2.0	1.0	84.0
10.0	30.0	0.0	1.0	16.0	2.0	3.0	89.0
6.0	26.0	3.0	2.0	14.0	1.0	5.0	64.0
11.0	30.0	1.0	1.0	16.0	1.0	1.0	133.0
7.0	27.0	3.0	0.0	12.0	1.0	2.0	108.0
11.0	38.0	7.0	1.0	19.0	2.0	2.0	171.0

NUMBER OF DATA POINTS = 10

RESULTS:

ESTIMATE OF B

B1	B2
7.14706	0.01562
23.26596	0.06875
4.03958	-0.00241
1.50814	0.00093
9.71870	0.04810
1.54722	0.00053
1.93402	0.00066

ESTIMATE OF COVARIANCE MATRIX SIGMA

2.542	1.314	-1.098	-0.237	0.409	-0.075	-0.937
1.314	4.626	3.399	0.792	0.883	0.955	-0.650
-1.098	3.399	8.665	0.538	-2.176	0.658	-0.615
-0.237	0.792	0.538	0.795	0.482	0.172	-0.004
0.409	0.883	-2.176	0.492	4.619	0.471	0.808
-0.075	0.955	0.658	0.172	0.471	0.548	-0.252
-0.937	-0.650	-0.615	-0.004	0.808	-0.252	1.747

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
2.15173	REJECT
7.01834	REJECT
-0.17978	ACCEPT
0.22784	ACCEPT
4.91423	REJECT
0.15746	ACCEPT
0.11025	ACCEPT

ENGINE NUMBER: 515355

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
10.0	39.0	5.0	0.0	17.0	2.0	4.0	126.0
9.0	29.0	2.0	1.0	11.0	2.0	3.0	117.0
12.0	31.0	3.0	0.0	14.0	2.0	2.0	84.0
6.0	23.0	0.0	2.0	10.0	0.0	0.0	66.0
5.0	19.0	9.0	0.0	7.0	0.0	1.0	24.0
4.0	20.0	3.0	1.0	7.0	1.0	2.0	10.0
8.0	33.0	1.0	0.0	15.0	2.0	1.0	79.0
5.0	26.0	6.0	2.0	12.0	1.0	1.0	46.0
10.0	28.0	9.0	3.0	11.0	2.0	4.0	19.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
5.19364	0.03898
19.83499	0.11819
6.56766	-0.03697
1.73114	-0.01152
7.73936	0.06015
0.68442	0.01023
1.43482	0.00891

ESTIMATE OF COVARIANCE MATRIX SIGMA

5.787	6.406	2.578	0.336	2.786	1.337	2.013
6.406	20.919	4.305	0.466	10.397	2.700	3.444
2.578	4.305	9.461	-0.004	1.620	0.526	2.665
0.336	0.466	-0.004	1.160	0.115	0.095	0.350
2.786	10.397	1.620	0.115	5.864	1.090	0.774
1.337	2.700	0.526	0.095	1.090	0.646	0.816
2.013	3.444	2.665	0.350	0.774	0.816	2.125

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.92706	REJECT
3.07322	REJECT
-1.42941	ACCEPT
-1.27241	ACCEPT
2.95414	REJECT
1.51371	ACCEPT
0.72672	ACCEPT

ENGINE NUMBER: 516678

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
9.0	26.0	5.0	1.0	13.0	3.0	6.0	60.0
8.0	25.0	7.0	1.0	12.0	1.0	5.0	89.0
8.0	25.0	3.0	1.0	11.0	1.0	1.0	79.0
7.0	20.0	2.0	0.0	14.0	2.0	5.0	58.0
10.0	35.0	5.0	0.0	14.0	3.0	3.0	29.0
8.0	31.0	1.0	0.0	12.0	3.0	4.0	31.0
8.0	30.0	8.0	1.0	12.0	4.0	3.0	117.0
4.0	26.0	5.0	2.0	10.0	2.0	3.0	51.0
5.0	20.0	9.0	1.0	8.0	1.0	0.0	18.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
6.73207	0.01205
26.23830	0.00537
3.94117	0.01791
0.40374	0.00633
11.05323	0.01226
1.85873	0.00615
2.48870	0.01429

ESTIMATE OF COVARIANCE MATRIX SIGMA

3.864	6.322	-1.535	-0.961	2.956	0.930	1.468
6.322	27.427	-2.397	-1.023	4.368	3.946	0.959
-1.535	-2.397	7.915	0.869	-3.111	-0.413	-2.296
-0.961	-1.023	0.869	0.462	-1.010	-0.267	-0.438
2.956	4.368	-3.111	-1.010	4.048	1.119	2.893
0.930	3.946	-0.413	-0.267	1.119	1.321	0.803
1.468	0.959	-2.296	-0.438	2.893	0.803	4.050

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
0.55162	ACCEPT
0.09220	ACCEPT
0.57286	ACCEPT
0.83792	ACCEPT
0.54810	ACCEPT
0.43131	ACCEPT
0.63885	ACCEPT

ENGINE NUMBER: 515058

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
10.0	23.0	7.0	1.0	8.0	0.0	2.0	26.0
7.0	25.0	2.0	0.0	10.0	2.0	3.0	139.0
4.0	20.0	6.0	0.0	8.0	1.0	2.0	83.0
17.	23.0	0.0	0.0	7.0	2.0	1.0	73.0
6.0	24.0	5.0	1.0	9.0	1.0	1.0	107.0
7.0	20.0	7.0	1.0	9.0	0.0	0.0	58.0
6.0	15.0	3.0	1.0	6.0	1.0	3.0	44.0
5.0	26.0	8.0	0.0	7.0	0.0	1.0	58.0
5.0	18.0	3.0	1.0	4.0	0.0	0.0	88.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
8.93460	-0.01984
20.24829	0.03220
7.14537	-0.03448
0.98016	-0.00565
6.16235	0.01855
-0.29047	0.01422
1.21374	0.00307

ESTIMATE OF COVARIANCE MATRIX SIGMA

17.509	18.182	-7.369	-0.609	0.886	2.073	-0.173
18.182	30.052	-3.002	-1.663	2.588	1.725	-0.941
-7.369	-3.002	6.738	0.059	2.024	-1.333	-0.605
-0.609	-1.663	0.059	0.275	-0.115	-0.163	-0.152
0.886	2.588	2.024	-0.115	3.289	0.237	0.607
2.073	1.725	-1.333	-0.163	0.237	0.525	0.498
-0.173	-0.941	-0.605	-0.152	0.607	0.498	1.448

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
-0.45716	ACCEPT
0.55631	ACCEPT
-1.28071	ACCEPT
-1.03937	ACCEPT
0.98615	ACCEPT
1.89257	ACCEPT
0.24613	ACCEPT

ENGINE NUMBER: 509090.

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
21.0	29.0	3.0	0.0	8.0	3.0	1.0	111.0
16.0	22.0	2.0	1.0	7.0	2.0	1.0	90.0
18.0	40.0	0.0	0.0	12.0	1.0	1.0	161.0
17.0	38.0	0.0	0.0	11.0	1.0	1.0	183.0
9.0	29.0	5.0	0.0	7.0	0.0	1.0	90.0
8.0	21.0	7.0	1.0	7.0	0.0	1.0	30.0
14.0	37.0	6.0	0.0	12.0	1.0	0.0	198.0
8.0	26.0	0.0	0.0	6.0	0.0	1.0	60.0
15.0	44.0	5.0	1.0	11.0	2.0	4.0	156.0
12.0	33.0	3.0	0.0	12.0	1.0	0.0	193.0
16.0	33.0	4.0	0.0	13.0	2.0	1.0	188.0

NUMBER OF DATA POINTS = 11

RESULTS:

ESTIMATE OF B

B1	B2
9.01636	0.03755
18.28833	0.10673
3.84371	-0.00499
0.73897	-0.00351
4.11884	0.04157
0.37839	0.01605
1.30356	-0.00160

ESTIMATE OF COVARIANCE MATRIX SIGMA

14.996	0.956	-3.165	0.176	-0.366	3.122	0.788
0.956	22.278	-0.155	0.297	1.499	-0.153	3.944
-3.165	-0.155	6.752	0.427	0.771	-0.035	0.282
0.176	0.297	0.427	0.195	0.130	0.133	0.281
-0.366	1.499	0.771	0.130	0.938	-0.114	0.187
3.122	-0.153	-0.035	0.133	-0.114	0.929	0.351
0.788	3.944	0.282	0.281	0.187	0.351	1.202

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.80834	ACCEPT
4.42040	REJECT
-0.35791	ACCEPT
-1.48459	ACCEPT
8.00668	REJECT
1.17122	ACCEPT
-0.27252	ACCEPT

ENGINE NUMBER: 524739

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
16.0	22.0	8.0	2.0	4.0	0.0	2.0	28.0
13.0	37.0	0.0	0.0	13.0	2.0	3.0	181.0
16.0	35.0	1.0	2.0	11.0	5.0	6.0	32.0
6.0	29.0	0.0	3.0	9.0	2.0	3.0	158.0
5.0	27.0	1.0	0.0	7.0	0.0	1.0	15.0
16.0	39.0	1.0	2.0	11.0	5.0	3.0	62.0
16.0	45.0	2.0	4.0	14.0	4.0	1.0	169.0
13.0	22.0	2.0	2.0	7.0	2.0	3.0	6.0
10.0	31.0	0.0	0.0	6.0	3.0	3.0	19.0
17.0	39.0	4.0	3.0	13.0	3.0	1.0	169.0
12.0	37.0	1.0	0.0	12.0	3.0	3.0	102.0
11.0	35.0	2.0	0.0	11.0	2.0	2.0	117.0
9.0	34.0	1.0	0.0	10.0	2.0	3.0	133.0
13.0	39.0	13.0	4.0	14.0	3.0	7.0	148.0

NUMBER OF DATA POINTS = 14

RESULTS:

ESTIMATE OF B

B1	B2
11.92093	0.00456
27.11922	0.06821
2.05082	0.00544
0.89575	0.00706
6.64235	0.03660
2.22275	0.00365
3.04211	-0.00119

ESTIMATE OF COVARIANCE MATRIX SIGMA

15.669	9.095	4.728	2.776	3.568	3.183	0.805
9.095	26.769	-1.434	0.710	8.839	6.396	1.686
4.728	-1.434	14.312	3.104	0.379	-0.975	2.828
2.776	0.710	3.104	2.383	0.515	0.747	0.671
3.568	8.839	0.379	0.515	4.309	2.357	1.551
3.183	6.396	-0.975	0.747	2.357	2.390	1.151
0.805	1.686	2.828	0.671	1.551	1.151	3.237

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
0.27445	ACCEPT
3.14026	REJECT
0.34273	ACCEPT
1.09012	ACCEPT
4.19994	REJECT
0.56178	ACCEPT
-0.15716	ACCEPT

ENGINE NUMBER: 516309

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
7.0	31.0	0.0	0.0	10.0	0.0	3.0	63.0
7.0	20.0	8.0	1.0	8.0	0.0	2.0	32.0
6.0	19.0	2.0	0.0	9.0	0.0	1.0	26.0
6.0	21.0	0.0	0.0	9.0	0.0	0.0	37.0
5.0	31.0	0.0	0.0	7.0	1.0	1.0	64.0
5.0	26.0	6.0	0.0	9.0	1.0	2.0	90.0
15.0	58.0	5.0	0.0	17.0	4.0	0.0	119.0
3.0	20.0	8.0	2.0	7.0	0.0	0.0	17.0
4.0	12.0	0.0	1.0	4.0	0.0	0.0	32.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
2.44113	0.07506
8.17088	0.34263
3.03289	0.00355
1.11980	-0.01266
4.70600	0.07843
-1.14859	0.03404
0.74821	0.00472

ESTIMATE OF COVARIANCE MATRIX SIGMA

6.415	13.140	1.098	-0.017	5.261	1.152	-0.747
13.140	49.284	3.719	0.820	13.528	3.445	-2.533
1.098	3.719	14.206	1.646	2.384	0.795	0.121
-0.017	0.820	1.646	0.395	-0.073	0.180	-0.208
5.261	13.528	2.384	-0.073	6.123	0.998	-0.482
1.152	3.445	0.795	0.180	0.998	0.493	-0.638
-0.747	-2.533	0.121	-0.208	-0.482	-0.638	1.400

T TEST OF THE HYPOTHESIS THAT A

COMPONENT OF THE B2 VECTOR EQUALS

ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
2.82837	REJECT
4.65781	REJECT
0.08989	ACCEPT
-1.92397	REJECT
3.02476	REJECT
4.62768	REJECT
0.38086	ACCEPT

ENGINE NUMBER: 516094

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
5.0	16.0	3.0	0.0	10.0	1.0	1.0	18.0
10.0	34.0	7.0	1.0	11.0	2.0	3.0	94.0
7.0	29.0	6.0	2.0	8.0	1.0	0.0	62.0
6.0	23.0	8.0	2.0	8.0	2.0	4.0	93.0
7.0	21.0	1.0	0.0	8.0	1.0	1.0	58.0
8.0	32.0	9.0	0.0	10.0	1.0	3.0	88.0
11.0	30.0	9.0	0.0	7.0	1.0	2.0	84.0
4.0	27.0	7.0	1.0	16.0	2.0	6.0	84.0
7.0	24.0	0.0	0.0	5.0	1.0	1.0	54.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
4.62017	0.03688
12.39392	0.20229
-1.43774	0.09912
-0.24837	0.01297
7.18618	0.02886
0.49376	0.01190
-1.03911	0.04780

ESTIMATE OF COVARIANCE MATRIX SIGMA

4.682	3.210	0.237	-0.817	-4.107	-0.551	-2.494
3.210	6.565	1.813	-0.012	-1.492	-0.429	-2.173
0.237	1.813	6.177	0.322	2.375	-0.078	0.530
-0.817	-0.012	0.322	0.737	0.257	0.176	-0.013
-4.107	-1.492	2.375	0.257	10.486	0.803	3.922
-0.551	-0.429	-0.078	0.176	0.803	0.185	0.452
-2.494	-2.173	0.530	-0.013	3.922	0.452	2.373

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.20353	ACCEPT
5.57493	REJECT
2.81619	REJECT
1.06650	ACCEPT
0.62926	ACCEPT
1.95434	REJECT
2.19124	REJECT

ENGINE NUMBER: 516949

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
5.0	22.0	1.0	1.0	10.0	2.0	5.0	55.0
7.0	19.0	8.0	2.0	7.0	2.0	2.0	8.0
12.0	21.0	6.0	0.0	8.0	3.0	2.0	29.0
8.0	27.0	7.0	0.0	10.0	3.0	2.0	50.0
6.0	17.0	2.0	0.0	7.0	1.0	2.0	55.0
5.0	24.0	8.0	2.0	10.0	2.0	5.0	85.0
5.0	19.0	4.0	0.0	6.0	2.0	3.0	39.0
6.0	29.0	0.0	3.0	13.0	3.0	6.0	86.0
5.0	26.0	3.0	1.0	11.0	2.0	4.0	95.0
8.0	43.0	2.0	1.0	14.0	1.0	1.0	104.0

NUMBER OF DATA POINTS = 10

RESULTS:

ESTIMATE OF B

B1	B2
8.14782	-0.02389
14.23561	0.17268
6.89968	-0.04620
0.36023	0.01056
5.36919	0.06982
2.47969	-0.00627
2.07484	0.01857

ESTIMATE OF COVARIANCE MATRIX SIGMA

4.891	7.131	0.960	-0.600	1.667	0.499	-1.942
7.131	30.781	1.478	0.139	6.593	0.216	-5.292
0.960	1.478	7.538	0.156	-0.312	0.297	-0.841
-0.600	0.139	0.156	1.129	0.697	0.197	1.036
1.667	6.593	-0.312	0.697	2.491	0.526	-0.062
0.499	0.216	0.297	0.197	0.526	0.570	0.602
-1.942	-5.292	-0.841	1.036	-0.062	0.602	2.824

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
-1.00850	ACCEPT
2.90546	REJECT
-1.57087	ACCEPT
0.92769	ACCEPT
4.12975	REJECT
-0.77488	ACCEPT
1.03131	ACCEPT

ENGINE NUMBER: 516750

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
5.0	11.0	3.0	0.0	12.0	2.0	2.0	21.0
6.0	9.0	2.0	0.0	12.0	1.0	2.0	21.0
6.0	13.0	6.0	1.0	13.0	3.0	3.0	27.0
8.0	28.0	7.0	1.0	12.0	1.0	5.0	61.0
3.0	13.0	2.0	0.0	4.0	0.0	1.0	86.0
4.0	16.0	6.0	1.0	10.0	1.0	4.0	13.0
5.0	15.0	2.0	1.0	10.0	0.0	0.0	15.0
10.0	34.0	8.0	2.0	12.0	1.0	3.0	54.0
8.0	17.0	6.0	0.0	12.0	2.0	3.0	22.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
5.92833	0.00514
12.54119	0.13478
4.35794	0.00868
0.59294	0.00207
13.03024	-0.06335
1.74137	-0.01463
2.42961	0.00354

ESTIMATE OF COVARIANCE MATRIX SIGMA

5.536	15.443	4.301	0.897	4.700	0.738	1.764
15.443	64.659	16.711	4.937	11.656	0.208	6.840
4.301	16.711	6.516	1.272	4.023	1.188	3.358
0.897	4.937	1.272	0.568	0.716	-0.025	0.376
4.700	11.656	4.023	0.716	5.559	1.384	2.324
0.738	0.208	1.188	-0.025	1.384	0.923	0.736
1.764	6.840	3.358	0.376	2.324	0.736	2.594

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
0.15670	ACCEPT
1.20225	ACCEPT
0.24397	ACCEPT
0.19728	ACCEPT
-1.92719	REJECT
-1.09022	ACCEPT
0.15775	ACCEPT

ENGINE NUMBER: 516575

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
7.0	36.0	0.0	0.0	8.0	1.0	6.0	22.0
5.0	37.0	0.0	0.0	12.0	2.0	2.0	66.0
6.0	36.0	3.0	0.0	11.0	2.0	2.0	58.0
17.0	28.0	4.0	0.0	9.0	2.0	1.0	28.0
16.0	35.0	8.0	1.0	8.0	5.0	4.0	36.0
16.0	50.0	6.0	2.0	17.0	4.0	4.0	112.0
10.0	39.0	8.0	1.0	10.0	4.0	4.0	85.0
13.0	34.0	2.0	0.0	9.0	5.0	3.0	63.0
5.0	25.0	7.0	1.0	6.0	0.0	0.0	27.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
8.78760	0.03202
24.76978	0.19532
3.14465	0.01951
-0.16926	0.01313
5.01866	0.09021
1.05098	0.03127
2.18423	0.01276

ESTIMATE OF COVARIANCE MATRIX SIGMA

27.819	3.220	6.475	1.022	1.557	5.828	1.510
3.220	16.504	-3.422	0.356	3.898	0.785	6.478
6.475	-3.422	11.253	1.717	-2.551	1.427	-1.085
1.022	0.356	1.717	0.423	-0.093	0.016	0.047
1.557	3.898	-2.551	-0.093	2.936	-0.801	0.227
5.828	0.785	1.427	0.016	-0.801	2.630	1.123
1.510	6.478	-1.085	0.047	0.227	1.123	3.671

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
0.51881	ACCEPT
4.10930	REJECT
0.49718	ACCEPT
1.72416	ACCEPT
4.49938	REJECT
1.64798	ACCEPT
0.55922	ACCEPT

ENGINE NUMBER: 516534

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
11.0	27.0	5.0	0.0	8.0	1.0	4.0	17.0
14.0	32.0	7.0	1.0	14.0	1.0	3.0	19.0
15.0	43.0	6.0	0.0	17.0	2.0	4.0	80.0
15.0	42.0	7.0	2.0	15.0	3.0	1.0	84.0
14.0	28.0	7.0	3.0	8.0	2.0	3.0	24.0
18.0	27.0	1.0	3.0	7.0	2.0	2.0	24.0
14.0	34.0	10.0	1.0	9.0	2.0	6.0	9.0
17.0	39.0	1.0	0.0	15.0	2.0	5.0	54.0
14.0	38.0	8.0	0.0	10.0	2.0	3.0	32.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
13.79272	0.02293
27.26935	0.18827
6.41499	-0.01672
1.36354	-0.00662
7.34557	0.10755
1.36015	0.01387
4.13487	-0.01812

ESTIMATE OF COVARIANCE MATRIX SIGMA

4.105	-0.638	-3.898	1.182	-0.425	0.385	-0.298
-0.638	14.603	6.490	-3.101	5.078	0.176	3.199
-3.898	6.490	10.260	-0.495	1.293	0.317	0.430
1.182	-3.101	-0.495	1.802	-1.718	0.383	-1.313
-0.425	5.078	1.293	-1.718	6.061	-0.688	1.617
0.385	0.176	0.317	0.383	-0.688	0.242	-0.142
-0.298	3.199	0.430	-1.313	1.617	-0.142	2.312

T TEST OF THE HYPOTHESIS THAT A

COMPONENT OF THE B2 VECTOR EQUALS

ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
0.89167	ACCEPT
3.88142	REJECT
-0.41124	ACCEPT
-0.38869	ACCEPT
3.44178	REJECT
2.22170	REJECT
-0.93863	ACCEPT

ENGINE NUMBER: 509081

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
4.0	17.0	6.0	1.0	5.0	1.0	1.0	57.0
19.0	29.0	1.0	0.0	1.0	3.0	1.0	91.0
2.0	19.0	0.0	1.0	4.0	0.0	0.0	63.0
7.0	32.0	5.0	0.0	9.0	2.0	0.0	180.0
8.0	39.0	9.0	0.0	9.0	3.0	2.0	209.0
5.0	21.0	0.0	0.0	5.0	0.0	1.0	237.0
7.0	27.0	4.0	0.0	5.0	3.0	1.0	148.0
3.0	26.0	6.0	0.0	7.0	1.0	1.0	110.0
6.0	21.0	8.0	0.0	6.0	1.0	0.0	118.0
9.0	24.0	2.0	0.0	6.0	2.0	2.0	106.0

NUMBER OF DATA POINTS = 10

RESULTS:

ESTIMATE OF B

B1	B2
7.02002	-0.00015
17.74504	0.05879
3.24261	0.00650
0.77586	-0.00437
3.09820	0.01973
1.23718	0.00275
0.55600	0.00261

ESTIMATE OF COVARIANCE MATRIX SIGMA

25.500	15.287	-3.121	-1.003	-5.738	4.377	1.127
15.287	35.331	7.989	-0.818	3.663	5.959	1.431
-3.121	7.989	11.689	-0.158	5.135	1.226	0.193
-1.003	-0.818	-0.158	0.122	0.055	-0.226	-0.053
-5.738	3.663	5.135	0.055	4.661	0.002	0.001
4.377	5.959	1.226	-0.226	0.002	1.519	0.420
1.127	1.431	0.193	-0.053	0.001	0.420	0.584

T TEST OF THE HYPOTHESIS THAT A

COMPONENT OF THE B2 VECTOR EQUALS

ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
-0.00546	ACCEPT
1.79515	ACCEPT
0.34506	ACCEPT
-2.27294	REJECT
1.65826	ACCEPT
0.40507	ACCEPT
0.61910	ACCEPT

ENGINE NUMBER: 516592

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
9.0	21.0	0.0	0.0	12.0	2.0	4.0	89.0
3.0	11.0	5.0	1.0	6.0	2.0	2.0	30.0
5.0	22.0	4.0	0.0	10.0	4.0	3.0	59.0
5.0	19.0	5.0	1.0	9.0	3.0	2.0	83.0
6.0	17.0	2.0	1.0	10.0	2.0	4.0	95.0
9.0	18.0	0.0	0.0	11.0	3.0	5.0	63.0
7.0	15.0	6.0	1.0	8.0	2.0	4.0	37.0
6.0	16.0	2.0	1.0	10.0	2.0	0.0	33.0
6.0	13.0	4.0	2.0	4.0	1.0	0.0	34.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
4.66621	0.02678
13.20392	0.07297
5.39080	-0.03923
1.43099	-0.01124
5.22276	0.06309
1.73505	0.01030
0.43833	0.03835

ESTIMATE OF COVARIANCE MATRIX SIGMA

3.668	1.792	-2.649	-0.418	1.870	-0.308	1.302
1.792	8.204	-0.995	-0.525	1.222	0.944	-0.828
-2.649	-0.995	4.367	0.548	-2.358	0.264	-0.362
-0.418	-0.525	0.548	0.410	-1.055	-0.387	-0.619
1.870	1.222	-2.358	-1.055	4.196	0.832	1.227
-0.308	0.944	0.264	-0.387	0.832	0.775	0.409
1.302	-0.828	-0.362	-0.619	1.227	0.409	2.579

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.02799	ACCEPT
1.87332	ACCEPT
-1.38041	ACCEPT
-1.29033	ACCEPT
2.26479	REJECT
0.85980	ACCEPT
1.75595	ACCEPT

ENGINE NUMBER: 515931

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
18.0	35.0	0.0	1.0	11.0	2.0	3.0	60.0
10.0	27.0	0.0	0.0	10.0	3.0	2.0	36.0
8.0	34.0	3.0	0.0	11.0	2.0	3.0	31.0
10.0	37.0	7.0	1.0	14.0	2.0	3.0	120.0
8.0	30.0	5.0	0.0	10.0	2.0	2.0	89.0
12.0	34.0	6.0	1.0	11.0	0.0	5.0	89.0
5.0	23.0	5.0	0.0	8.0	0.0	1.0	18.0
10.0	33.0	2.0	1.0	11.0	0.0	1.0	79.0
6.0	27.0	0.0	0.0	8.0	0.0	2.0	51.0
5.0	21.0	4.0	0.0	7.0	0.0	1.0	9.0

NUMBER OF DATA POINTS = 10

RESULTS:

ESTIMATE OF B

B1	B2
6.43751	0.04747
23.34947	0.11599
1.50781	0.02908
-0.18435	0.01004
7.48360	0.04496
0.75608	0.00591
1.29868	0.01720

ESTIMATE OF COVARIANCE MATRIX SIGMA

13.719	8.704	-4.904	0.966	2.165	1.698	1.879
8.704	13.568	-2.737	0.655	3.884	1.755	2.225
-4.904	-2.737	6.488	-0.144	-0.150	-0.896	0.208
0.966	0.655	-0.144	0.155	0.178	-0.135	0.102
2.165	3.884	-0.150	0.178	1.714	0.982	0.478
1.698	1.755	-0.896	-0.135	0.982	1.562	0.192
1.879	2.225	0.208	0.102	0.478	0.192	1.338

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.37268	ACCEPT
3.37293	REJECT
1.22274	ACCEPT
2.72805	REJECT
3.67812	REJECT
0.50639	ACCEPT
1.59321	ACCEPT

ENGINE NUMBER: 515608

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
6.0	38.0	4.0	1.0	16.0	1.0	3.0	22.0
8.0	40.0	1.0	0.0	22.0	2.0	4.0	66.0
4.0	31.0	10.0	0.0	18.0	1.0	4.0	58.0
7.0	31.0	7.0	0.0	14.0	1.0	2.0	28.0
10.0	37.0	1.0	1.0	21.0	2.0	6.0	112.0
2.0	30.0	3.0	1.0	15.0	2.0	3.0	85.0
4.0	21.0	3.0	2.0	12.0	1.0	0.0	63.0
7.0	27.0	4.0	0.0	12.0	3.0	5.0	36.0
6.0	30.0	9.0	1.0	12.0	0.0	1.0	27.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
6.25582	-0.00061
30.13470	0.02774
8.04047	-0.06110
0.35927	0.00557
11.71527	0.07357
0.78598	0.01192
1.55565	0.02817

ESTIMATE OF COVARIANCE MATRIX SIGMA

7.079	10.684	-3.229	-0.473	4.967	0.452	2.700
10.684	39.197	-4.374	-2.018	18.775	0.274	6.375
-3.229	-4.374	8.390	-0.359	-1.119	-1.335	-0.585
-0.473	-2.018	-0.359	0.539	-1.380	-0.307	-0.973
4.967	18.775	-1.119	-1.380	11.145	0.069	3.155
0.452	0.274	-1.335	-0.307	0.069	0.741	1.015
2.700	6.375	-0.585	-0.973	3.155	1.015	3.299

T TEST OF THE HYPOTHESIS THAT A

COMPONENT OF THE B2 VECTOR EQUALS

ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
-0.01954	ACCEPT
0.37873	ACCEPT
-1.80280	ACCEPT
0.64801	ACCEPT
1.88347	ACCEPT
1.18436	ACCEPT
1.32551	ACCEPT

ENGINE NUMBER: 516380

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
5.0	19.0	6.0	0.0	9.0	1.0	4.0	60.0
10.0	35.0	3.0	1.0	15.0	1.0	1.0	58.0
18.0	43.0	5.0	2.0	13.0	2.0	5.0	52.0
4.0	24.0	7.0	2.0	10.0	1.0	1.0	12.0
6.0	27.0	9.0	1.0	10.0	1.0	1.0	29.0
6.0	25.0	1.0	1.0	12.0	1.0	1.0	41.0
5.0	26.0	6.0	0.0	12.0	1.0	2.0	66.0
5.0	19.0	7.0	3.0	8.0	1.0	5.0	6.0
4.0	16.0	5.0	0.0	4.0	1.0	0.0	39.0
5.0	15.0	5.0	1.0	5.0	1.0	0.0	31.0
3.0	19.0	2.0	0.0	6.0	1.0	2.0	18.0

NUMBER OF DATA POINTS = 11

RESULTS:

ESTIMATE OF B

B1	B2
3.26027	0.08528
17.74008	0.17684
5.94449	-0.02279
1.97730	-0.02609
6.24200	0.08577
0.95806	0.00355
1.87213	0.00341

ESTIMATE OF COVARIANCE MATRIX SIGMA

16.545	28.482	0.057	2.681	6.526	1.145	3.756
28.482	65.145	0.463	4.880	20.665	1.785	5.392
0.057	0.463	5.864	0.507	-0.049	0.027	0.924
2.681	4.880	0.507	0.801	1.909	0.153	0.929
6.526	20.665	-0.049	1.909	10.062	0.255	1.644
1.145	1.785	0.027	0.153	0.255	0.095	0.328
3.756	5.392	0.924	0.929	1.644	0.328	3.772

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.34267	ACCEPT
1.40306	ACCEPT
-0.60264	ACCEPT
-1.85709	REJECT
1.73153	ACCEPT
0.73587	ACCEPT
0.11256	ACCEPT

ENGINE NUMBER: 515515

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
18.0	41.0	7.0	0.0	13.0	2.0	6.0	42.0
18.0	45.0	1.0	0.0	11.0	3.0	5.0	55.0
17.0	45.0	3.0	0.0	11.0	2.0	4.0	58.0
12.0	36.0	9.0	1.0	12.0	2.0	4.0	37.0
12.0	37.0	7.0	0.0	10.0	2.0	1.0	24.0
10.0	32.0	3.0	1.0	8.0	2.0	0.0	24.0
9.0	34.0	1.0	1.0	13.0	1.0	3.0	62.0
18.0	43.0	2.0	0.0	15.0	2.0	5.0	76.0
11.0	36.0	6.0	0.0	8.0	2.0	2.0	3.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
10.12277	0.08896
32.50766	0.16124
7.40925	-0.07738
0.37390	-0.00096
7.57781	0.08609
2.[illegible]7099	-0.05168
0.97021	0.05582

ESTIMATE OF COVARIANCE MATRIX SIGMA

11.693	14.066	2.153	-1.473	0.465	1.375	3.944
14.066	20.498	1.440	-2.194	0.057	1.733	4.062
2.153	1.440	6.429	-0.044	2.591	-0.077	2.147
-1.473	-2.194	-0.044	0.285	-0.046	-0.144	-0.397
0.465	0.057	2.591	-0.046	1.803	-0.200	1.182
1.375	1.733	-0.077	-0.144	-0.200	0.284	0.342
3.944	4.062	2.147	-0.397	1.182	0.342	2.713

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.68080	ACCEPT
2.30081	REJECT
-1.97172	REJECT
-0.11594	ACCEPT
4.14212	REJECT
-0.20330	ACCEPT
2.18941	REJECT

ENGINE NUMBER: 516731

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
8.0	16.0	6.0	1.0	14.0	2.0	5.0	54.0
11.0	29.0	6.0	2.0	16.0	3.0	6.0	83.0
13.0	21.0	8.0	2.0	9.0	1.0	3.0	57.0
6.0	20.0	6.0	3.0	9.0	0.0	5.0	33.0
5.0	20.0	8.0	0.0	8.0	1.0	1.0	87.0
10.0	22.0	5.0	1.0	13.0	2.0	3.0	27.0
13.0	24.0	5.0	1.0	7.0	1.0	0.0	22.0
5.0	23.0	7.0	2.0	9.0	2.0	4.0	99.0
5.0	15.0	5.0	1.0	8.0	1.0	0.0	31.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
10.39656	-0.03564
18.45300	0.04852
4.60350	0.02955
1.45355	-0.00017
9.18830	0.02090
0.67471	0.01405
1.45051	0.02829

ESTIMATE OF COVARIANCE MATRIX SIGMA

11.962	9.873	0.450	0.312	3.521	1.224	1.248
9.873	17.879	-1.115	1.230	3.984	1.429	2.261
0.450	-1.115	0.817	0.021	-1.399	-0.523	-0.369
0.312	1.230	0.021	0.889	0.384	-0.109	1.433
3.521	3.984	-1.399	0.384	11.011	2.243	5.436
1.224	1.429	-0.523	-0.109	2.243	0.700	0.621
1.248	2.261	-0.369	1.433	5.436	0.621	4.951

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
-0.84223	ACCEPT
0.93805	ACCEPT
2.67191	REJECT
-0.01442	ACCEPT
0.51491	ACCEPT
1.37245	ACCEPT
1.03920	ACCEPT

ENGINE NUMBER: 515447

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
10.0	32.0	9.0	0.0	11.0	1.0	2.0	65.0
12.0	41.0	2.0	3.0	15.0	2.0	2.0	82.0
8.0	25.0	4.0	0.0	9.0	6.0	0.0	46.0
7.0	28.0	4.0	1.0	11.0	1.0	2.0	54.0
8.0	25.0	1.0	2.0	10.0	1.0	3.0	60.0
5.0	27.0	3.0	1.0	9.0	1.0	0.0	71.0
4.0	21.0	4.0	0.0	8.0	1.0	0.0	26.0
6.0	27.0	7.0	1.0	9.0	1.0	3.0	26.0
12.0	39.0	3.0	0.0	12.0	5.0	2.0	82.0
12.0	25.0	2.0	0.0	8.0	4.0	0.0	70.0

NUMBER OF DATA POINTS = 10

RESULTS:

ESTIMATE OF B

B1	B2
1.99201	0.11010
16.25591	0.22241
6.64628	-0.04719
-0.00295	0.01380
6.42582	0.06485
0.84094	0.02507
1.25628	0.00247

ESTIMATE OF COVARIANCE MATRIX SIGMA

4.404	3.621	0.845	-0.357	0.825	2.065	0.673
3.621	21.914	4.162	1.496	7.358	0.203	3.519
0.845	4.162	5.576	-0.847	0.825	-0.787	0.729
-0.357	1.496	-0.847	1.111	1.133	-1.086	0.709
0.825	7.358	0.825	1.133	3.242	-0.832	1.450
2.065	0.203	-0.787	-1.086	-0.832	3.970	-1.054
0.673	3.519	0.729	0.709	1.450	-1.054	1.797

T TEST OF THE HYPOTHESIS THAT A

COMPONENT OF THE B2 VECTOR EQUALS

ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
3.20222	REJECT
2.89991	REJECT
-1.21977	ACCEPT
0.79880	ACCEPT
2.19847	REJECT
0.76801	ACCEPT
0.11243	ACCEPT

ENGINE NUMBER: 516048.0

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
6.0	24.0	1.0	0.0	8.0	0.0	2.0	30.0
9.0	45.0	4.0	2.0	12.0	2.0	3.0	120.0
4.0	24.0	6.0	1.0	6.0	0.0	0.0	90.0
9.0	46.0	5.0	1.0	13.0	2.0	4.0	96.0
4.0	19.0	3.0	2.0	5.0	1.0	1.0	59.0
6.0	39.0	6.0	1.0	11.0	1.0	1.0	55.0
4.0	29.0	3.0	2.0	9.0	1.0	3.0	33.0
10.0	52.0	8.0	1.0	16.0	1.0	2.0	96.0
2.0	29.0	0.0	0.0	8.0	1.0	0.0	29.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
1.94411	0.06004
19.05020	0.22459
0.53192	0.05134
0.48330	0.00929
6.19133	0.05309
0.29463	0.01044
0.83647	0.01393

ESTIMATE OF COVARIANCE MATRIX SIGMA

4.123	14.290	0.952	-0.304	5.385	0.319	1.901
14.290	92.544	5.428	-1.773	30.546	3.206	6.096
0.952	5.428	3.967	0.088	2.277	-0.418	-0.368
-0.304	-1.773	0.088	0.585	-0.616	0.158	0.290
5.385	30.546	2.277	-0.616	10.520	0.843	2.251
0.319	3.206	-0.418	0.158	0.843	0.428	0.523
1.901	6.096	-0.368	0.290	2.251	0.523	1.967

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
2.83524	REJECT
2.23854	REJECT
2.47146	REJECT
1.16508	ACCEPT
1.56945	ACCEPT
1.52994	ACCEPT
0.95259	ACCEPT

ENGINE NUMBER: 516063

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
2.0	11.0	3.0	0.0	6.0	6.0	2.0	0.0
7.0	20.0	0.0	0.0	12.0	1.0	4.0	71.0
6.0	18.0	2.0	0.0	10.0	1.0	2.0	34.0
8.0	19.0	0.0	1.0	9.0	1.0	4.0	40.0
2.0	13.0	8.0	1.0	8.0	0.0	2.0	27.0
9.0	23.0	1.0	0.0	14.0	1.0	2.0	0.0
4.0	17.0	9.0	1.0	9.0	2.0	4.0	24.0
5.0	20.0	5.0	1.0	9.0	7.0	2.0	17.0
7.0	32.0	2.0	0.0	14.0	7.0	4.0	65.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
4.53252	0.03312
15.36964	0.12472
4.63345	-0.04209
0.53999	-0.00309
8.66085	0.24695
3.11069	-0.00718
1.94980	0.03040

ESTIMATE OF COVARIANCE MATRIX SIGMA

6.387	9.302	-5.951	-0.387	5.089	-1.321	0.356
9.302	30.475	-5.324	-0.850	12.190	6.961	0.450
-5.951	-5.324	11.299	1.145	-3.342	-0.027	0.253
-0.387	-0.850	1.145	0.311	-0.673	-0.238	0.131
5.089	12.190	-3.342	-0.673	6.829	-0.170	-0.010
-1.321	6.961	-0.027	-0.238	-0.170	9.519	-0.002
0.356	0.450	0.253	0.131	-0.010	-0.002	0.606

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
0.92937	ACCEPT
1.60218	ACCEPT
-0.88798	ACCEPT
-0.39360	ACCEPT
1.27409	ACCEPT
-0.16505	ACCEPT
2.76992	REJECT

ENGINE NUMBER: 509064

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
14.0	45.0	1.0	1.0	22.0	2.0	4.0	240.0
11.0	31.0	2.0	1.0	11.0	3.0	5.0	103.0
9.0	36.0	6.0	0.0	13.0	4.0	2.0	90.0
10.0	35.0	7.0	1.0	11.0	4.0	4.0	67.0
10.0	24.0	7.0	1.0	8.0	5.0	1.0	30.0
14.0	42.0	8.0	2.0	19.0	2.0	6.0	210.0
14.0	43.0	8.0	3.0	19.0	2.0	6.0	217.0
11.0	35.0	9.0	0.0	13.0	2.0	5.0	150.0
8.0	31.0	5.0	1.0	10.0	0.0	3.0	60.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
7.72752	0.02695
25.42332	0.07985
6.27236	-0.00297
0.36484	0.00576
6.15189	0.06053
3.73031	-0.00820
1.88632	0.01630

ESTIMATE OF COVARIANCE MATRIX SIGMA

0.990	-1.163	0.290	0.483	-0.108	0.886	0.151
-1.163	8.229	0.152	-0.098	2.517	-0.063	-0.150
0.290	0.152	8.639	0.561	-1.064	0.358	1.043
0.483	-0.098	0.561	0.759	0.057	-0.059	0.361
-0.108	2.517	-1.064	0.057	1.626	0.381	-1.004
0.886	-0.063	0.358	-0.059	0.381	2.113	-0.518
0.151	-0.150	1.043	0.361	-1.004	-0.518	1.619

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
5.91299	REJECT
6.07753	REJECT
-0.22025	ACCEPT
1.44270	ACCEPT
10.36179	REJECT
-1.23196	ACCEPT
2.79686	REJECT

ENGINE NUMBER: 524646

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
12.0	38.0	2.0	0.0	11.0	2.0	4.0	32.0
9.0	40.0	7.0	0.0	13.0	2.0	6.0	32.0
10.0	40.0	1.0	2.0	11.0	2.0	5.0	27.0
9.0	42.0	0.0	0.0	9.0	3.0	2.0	22.0
8.0	30.0	2.0	0.0	10.0	1.0	1.0	9.0
7.0	28.0	0.0	1.0	8.0	1.0	1.0	8.0
11.0	42.0	3.0	0.0	12.0	2.0	5.0	63.0
11.0	45.0	6.0	0.0	12.0	2.0	3.0	72.0
15.0	56.0	0.0	0.0	19.0	5.0	7.0	126.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
7.78304	0.05629
31.59843	0.19645
2.17532	0.00365
0.58916	-0.00590
8.38977	0.07562
1.10359	0.02581
2.06722	0.03947

ESTIMATE OF COVARIANCE MATRIX SIGMA

1.295	1.345	-0.719	0.023	0.377	0.260	0.693
1.345	13.200	0.202	0.146	0.747	1.910	2.558
-0.719	0.202	7.692	-0.679	0.546	-1.107	1.287
0.023	0.146	-0.679	0.514	0.020	0.013	0.336
0.377	0.747	0.546	0.020	2.019	0.312	1.564
0.260	1.910	-1.107	0.013	0.312	0.554	0.244
0.693	2.558	1.287	0.336	1.564	0.244	2.801

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
5.30951	REJECT
5.80300	REJECT
0.14110	ACCEPT
-0.88370	ACCEPT
5.71138	REJECT
3.72125	REJECT
2.53127	REJECT

ENGINE NUMBER: 5166270

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
8.0	32.0	9.0	0.0	7.0	3.0	3.0	24.0
4.0	18.0	1.0	0.0	7.0	3.0	3.0	10.0
11.0	37.0	9.0	2.0	14.0	3.0	4.0	86.0
9.0	27.0	2.0	0.0	9.0	2.0	4.0	24.0
5.0	23.0	2.0	1.0	7.0	1.0	0.0	3.0
10.0	32.0	3.0	0.0	13.0	2.0	3.0	63.0
10.0	39.0	2.0	0.0	13.0	2.0	0.0	68.0
8.0	35.0	0.0	0.0	13.0	2.0	3.0	68.0
6.0	28.0	2.0	1.0	11.0	1.0	1.0	23.0
6.0	27.0	2.0	0.0	11.0	1.0	1.0	36.0
6.0	30.0	2.0	0.0	12.0	3.0	4.0	60.0

NUMBER OF DATA POINTS = 11

RESULTS:

ESTIMATE OF B

B1	B2
4.95470	0.06129
21.82213	0.18915
2.09675	0.02352
0.20373	0.00378
6.62466	0.09017
1.67202	0.00991
1.61738	0.01765

ESTIMATE OF COVARIANCE MATRIX SIGMA

2.669	3.278	2.382	0.116	-0.339	-0.021	0.172
3.278	11.578	3.910	0.029	-0.452	-0.571	-2.088
2.382	3.910	9.631	0.995	-1.871	1.125	1.274
0.116	0.029	0.995	0.493	0.094	-0.072	-0.107
-0.339	-0.452	-1.871	0.094	1.157	-0.718	-0.746
-0.021	-0.571	1.125	-0.072	-0.718	0.684	0.922
0.172	-2.088	1.274	-0.107	-0.746	0.922	2.463

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
3.27932	REJECT
4.85956	REJECT
0.66246	ACCEPT
0.47103	ACCEPT
7.32913	REJECT
1.04718	ACCEPT
0.98340	ACCEPT

ENGINE NUMBER: 516101

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
16.0	44.0	3.0	0.0	23.0	3.0	4.0	80.0
16.0	40.0	0.0	0.0	23.0	1.0	1.0	128.0
5.0	17.0	1.0	3.0	8.0	0.0	4.0	128.0
10.0	33.0	9.0	1.0	19.0	1.0	1.0	192.0
11.0	36.0	3.0	0.0	21.0	1.0	2.0	114.0
8.0	29.0	9.0	0.0	17.0	1.0	0.0	56.0
10.0	37.0	1.0	0.0	16.0	2.0	3.0	90.0
7.0	21.0	5.0	2.0	10.0	1.0	0.0	23.0
6.0	24.0	2.0	0.0	11.0	1.0	1.0	23.0
7.0	22.0	8.0	2.0	9.0	1.0	4.0	11.0
11.0	25.0	7.0	0.0	15.0	1.0	1.0	35.0

NUMBER OF DATA POINTS = 11

RESULTS:

ESTIMATE OF B

B1	B2
8.01412	0.02141
24.55724	0.06576
5.14212	-0.00973
0.71736	0.00012
11.69686	0.04924
1.27603	-0.00118
1.73059	0.00223

ESTIMATE OF COVARIANCE MATRIX SIGMA

13.487	26.779	-2.465	-2.767	16.654	2.040	0.132
26.779	69.565	-6.069	-7.645	38.864	5.651	0.676
-2.465	-6.069	12.388	0.348	-0.676	-0.344	-1.771
-2.767	-7.645	0.348	1.353	-4.921	-0.494	0.635
16.654	38.864	-0.676	-4.921	25.366	2.733	-1.545
2.040	5.651	-0.344	-0.494	2.733	0.621	0.363
0.132	0.676	-1.771	0.635	-1.545	0.363	2.750

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.04745	ACCEPT
1.41632	ACCEPT
-0.49665	ACCEPT
0.01914	ACCEPT
1.75634	ACCEPT
-0.26838	ACCEPT
0.24171	ACCEPT

ENGINE NUMBER: 516132

DATA:

Y1	Y2	Y3	Y4	Y5	Y6	Y7	X1
7.0	25.0	9.0	2.0	8.0	1.0	2.0	26.0
16.0	29.0	3.0	0.0	9.0	2.0	0.0	100.0
9.0	35.0	6.0	2.0	11.0	2.0	4.0	139.0
5.0	31.0	7.0	1.0	9.0	2.0	2.0	73.0
9.0	31.0	4.0	0.0	9.0	2.0	0.0	107.0
10.0	29.0	7.0	0.0	11.0	1.0	0.0	58.0
6.0	23.0	3.0	0.0	7.0	2.0	2.0	44.0
7.0	24.0	2.0	0.0	7.0	1.0	1.0	58.0
7.0	20.0	10.0	0.0	5.0	1.0	2.0	35.0

NUMBER OF DATA POINTS = 9

RESULTS:

ESTIMATE OF B

B1	B2
5.56531	0.04049
19.68764	0.10908
7.67937	-0.02830
0.25305	0.00425
6.02439	0.03403
0.87049	0.00963
1.15398	0.00408

ESTIMATE OF COVARIANCE MATRIX SIGMA

9.420	-1.576	-1.412	-1.020	0.551	-0.225	-2.660
-1.576	6.789	2.538	1.229	3.546	0.008	0.036
-1.412	2.538	7.866	1.430	0.868	-0.327	1.660
-1.020	1.229	1.430	0.860	0.452	-0.034	0.941
0.551	3.546	0.868	0.452	2.472	-0.126	-0.333
-0.225	0.008	-0.327	-0.034	-0.126	0.170	0.048
-2.660	0.036	1.660	0.941	-0.333	0.048	2.005

T TEST OF THE HYPOTHESIS THAT A
COMPONENT OF THE B2 VECTOR EQUALS
ZERO AT A .10 SIGNIFICANCE LEVEL

TEST STATISTIC	RESULT
1.39313	ACCEPT
4.42127	REJECT
-1.06572	ACCEPT
0.48443	ACCEPT
2.28583	REJECT
2.47053	REJECT
0.30463	ACCEPT

F. COMPARISON OF ESTIMATED PARAMETERS AMONG ENGINES

ESTIMATED PARAMETERS FOR CHROMIUM

SERIAL #	MEAN	VARIANCE
516726	3.8000	7.7333
515355	4.2222	10.6944
516678	5.0000	7.2500
515058	4.5556	7.2778
509090	3.1818	6.1636
524739	2.5714	13.3406
516309	3.2222	12.4444
516094	5.5556	11.5278
516949	4.1000	8.7667
516750	4.6667	5.7500
516575	4.2222	10.1944
516534	5.7778	9.1944
509081	4.1000	10.5444
516592	3.1111	4.8611
515931	3.2000	6.8444
515608	4.6667	10.7500
516380	5.0909	5.4909
515515	4.3333	8.7500
516731	6.2222	1.4444
515447	3.9000	5.8778
516048	4.0000	6.5000
516063	3.3333	11.0000
509064	5.8889	7.6111
524646	2.3333	6.7500
516627	3.0909	9.0909
516101	4.3636	11.4545
516132	5.6667	8.0000

ESTIMATED PARAMETERS FOR SILVER

SERIAL #	MEAN	VARIANCE
516726	1.6000	0.7111
515355	1.0000	1.2500
516678	0.7778	0.4444
515058	0.5556	0.2778
509090	0.2727	0.2182
524739	1.5714	2.4176
516309	0.4444	0.5278
516094	0.6667	0.7500
516949	1.0000	1.1111
516750	0.6667	0.5000
516575	0.5556	0.5278
516534	1.1111	1.6111
509081	0.2000	0.1778
516592	0.7778	0.4444
515931	0.4000	0.2667
515608	0.6667	0.5000
516380	1.0000	1.0000
515515	0.3333	0.2500
516731	1.4444	0.7778
515447	0.8000	1.0667
516048	1.1111	0.6111
516063	0.4444	0.2778
509064	1.1111	0.8611
524646	0.3333	0.5000
516627	0.3636	0.4545
516101	0.7273	1.2182
516132	0.5556	0.7778

ESTIMATED PARAMETERS FOR MAGNES.

SERIAL #	MEAN	VARIANCE
516726	1.6000	0.4889
515355	1.3333	0.7500
516678	2.2222	1.1944
515058	0.7778	0.6944
509090	1.1818	0.9636
524739	2.5714	2.2637
516309	0.6667	1.7500
516094	1.3333	0.2500
516949	2.1000	0.5444
516750	1.2222	0.9444
516575	2.7778	3.1944
516534	1.8889	0.3611
509081	1.6000	1.3778
516592	2.3333	0.7500
515931	1.1000	1.4333
515608	1.4444	0.7778
516380	1.0909	0.0909
515515	2.0000	0.2500
516731	1.4444	0.7778
515447	2.3000	3.7889
516048	1.0000	0.5000
516063	2.8889	8.3611
509064	2.6667	2.2500
524646	2.2222	1.4444
516627	2.0909	0.6909
516101	1.1818	0.5636
516132	1.5556	0.2778

ESTIMATED PARAMETERS FOR NICKEL

SERIAL #	MEAN	VARIANCE
516726	2.0000	1.5556
515355	2.0000	2.0000
516678	3.3333	3.7500
515058	1.4444	1.2778
509090	1.0909	1.0909
524739	2.9286	2.9945
516309	1.0000	1.2500
516094	2.3333	3.5000
516949	3.2000	2.8444
516750	2.5556	2.2778
516575	2.8889	3.3611
516534	3.4444	2.2778
509081	0.9000	0.5444
516592	2.6667	3.2500
515931	2.3000	1.5667
515608	3.1111	3.6111
516380	2.0000	3.4000
515515	3.3333	4.0000
516731	3.0000	5.0000
515447	1.4000	1.6000
516048	1.7778	1.9444
516063	2.8889	1.1111
509064	4.0000	3.0000
524646	3.7778	4.6944
516627	2.3636	2.4545
516101	1.9091	2.4909
516132	1.4444	1.7778

REGRESSION LINES FOR ALUMINUM

SERIAL #	INTERCEPT	SLOPE	RESIDUAL VARIANCE	SAMPLE CORRELATION
516726	8.7000	0.01562	2.5416	0.6055
515355	7.6667	0.03898	5.7870	0.5887
516678	7.4444	0.01205	3.8638	0.2041
515058	7.4444	-0.01984	17.5090	-0.1703
509090	14.0000	0.03755	14.9958	0.5162
524739	12.3571	0.00456	15.6695	0.0790
516309	6.4444	0.07506	6.4149	0.7303
516094	7.2222	0.03688	4.6820	0.4141
516949	6.7000	-0.02389	4.8907	-0.3358
516750	6.1111	0.00514	5.5361	0.0591
516575	10.5556	0.03202	27.8192	0.1924
516534	14.6667	0.02293	4.1051	0.3194
509081	7.0000	-0.00015	25.4999	-0.0019
516592	6.2222	0.02678	3.6684	0.3622
515931	9.2000	0.04747	13.7188	0.4366
515608	6.2222	-0.00061	7.0790	-0.0074
516380	6.4545	0.08528	16.5446	0.4085
515515	13.8889	0.08896	11.6934	0.5362
516731	8.4444	-0.03564	11.9624	-0.3033
515447	8.4000	0.11010	4.4045	0.7495
516048	6.0000	0.06004	4.1227	0.7311
516063	5.5556	0.03312	6.3866	0.3314
509064	11.2222	0.02695	0.9903	0.9128
524646	10.2222	0.05629	1.2945	0.8950
516627	7.5455	0.06129	2.6692	0.7378
516101	9.7273	0.02141	13.4872	0.3296
516132	8.4444	0.04049	9.4200	0.4659

REGRESSION LINES FOR IRON

SERIAL #	INTERCEPT	SLOPE	RESIDUAL VARIANCE	SAMPLE CORRELATION
516726	30.1000	0.06875	4.6265	0.9275
515355	27.3333	0.11819	20.9185	0.7578
516678	26.5555	0.00537	27.4270	0.0348
515058	22.6667	0.03220	30.0517	0.2093
509090	32.4545	0.10673	20.2778	0.8274
524739	33.6429	0.06821	26.7694	0.6716
516309	26.4444	0.34263	49.2843	0.8695
516094	26.6667	0.20229	6.5651	0.9034
516949	24.7000	0.17268	30.7814	0.7165
516750	17.3333	0.13478	64.6504	0.4137
516575	35.5555	0.19532	16.5041	0.8408
516534	34.4444	0.18827	14.6030	0.8263
509081	25.5000	0.05879	35.3305	0.5359
516592	17.4444	0.07297	8.2043	0.5779
515931	30.1000	0.11599	13.5679	0.7662
515608	31.6667	0.02774	39.1957	0.1417
516380	24.3636	0.17684	65.1446	0.4236
515515	39.3333	0.16124	20.4983	0.6562
516731	21.1111	0.04852	17.8794	0.3342
515447	29.2000	0.22241	21.9142	0.7159
516048	34.2222	0.22459	92.5444	0.6459
516063	19.2222	0.12472	30.4751	0.5180
509064	35.7778	0.07985	8.2291	0.9169
524646	40.1111	0.19645	13.1995	0.9099
516627	29.8182	0.18915	11.5785	0.8509
516101	29.8182	0.06576	69.5654	0.4269
516132	27.4444	0.10908	6.7886	0.8581

REGRESSION LINES FOR COPPER

SERIAL #	INTERCEPT	SLOPE	RESIDUAL VARIANCE	SAMPLE CORRELATION
516726	14.5000	0.04810	4.6190	0.8667
515355	11.5556	0.06015	5.8640	0.7449
516678	11.7778	0.01226	4.0485	0.2029
515058	7.5556	0.01855	3.2891	0.3493
509090	9.6364	0.04157	0.9376	0.9364
524739	10.1429	0.03660	4.3089	0.7714
516309	8.8889	0.07843	6.1235	0.7527
516094	9.2222	0.02886	10.4862	0.2314
516949	9.6000	0.06982	2.4905	0.8250
516750	10.7778	-0.06335	5.5586	-0.5888
516575	10.0000	0.09021	2.9364	0.8620
516534	11.4444	0.10755	6.0609	0.7928
509081	5.7000	0.01973	4.6605	0.5058
516592	8.8889	0.06309	4.1955	0.6503
515931	10.1000	0.04496	1.7140	0.7927
515608	15.7778	0.07357	11.1454	0.5799
516380	9.4545	0.08577	10.0621	0.4999
515515	11.2222	0.08609	1.8030	0.8428
516731	10.3333	0.02090	11.0115	0.1910
515447	10.2000	0.06485	3.2416	0.6137
516048	9.7778	0.05309	10.5203	0.5102
516063	10.1111	0.04695	6.8290	0.4339
509064	14.0000	0.06053	1.6263	0.9689
524646	11.6667	0.07562	2.0192	0.9074
516627	10.6364	0.09017	1.1567	0.9255
516101	15.6364	0.04924	25.3663	0.5052
516132	8.4444	0.03403	2.4722	0.6538

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13. ABSTRACT

This thesis examines spectrometric oil analysis data from two sources in an attempt to formulate a statistical model which will be useful in monitoring aircraft engines in the Naval Oil Analysis Program. Initially, experimental data, gathered for an Air Force study, is used to determine if the measurement error inherent in the monitoring procedure is normally distributed and if correlations exist between measurements for different wear metals. Based on the results of this investigation, a study is made of operational data from Wright reciprocating engines of the R1820-82 type. This investigation leads to the conclusion that a multivariate regression model is useful in estimating the parameters of the distribution of analyses from properly operating engines of this type. A procedure is then suggested which would employ the readings from past oil analyses from a particular engine to determine its present condition.

14 KEY WORDS	LINK A		LINK B		LINK C	
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MULTIVARIATE REGRESSION						
PREDICTION OF AIRCRAFT ENGINE WEAR						
MULTIVARIATE NORMAL CONFIDENCE REGIONS						
MULTIVARIATE NORMAL HYPOTHESIS TESTS						